Two AD-AS models: Neoclassical vs. Keynesian

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Abstract

Prior to the 'Lucas Critique', macroeconomic theory used to impose relationships among macroeconomic variables as model primitives. Two canonical examples of such frameworks are the neoclassical and the Keynesian AD-AS model. This note examines both of these models and concludes by discussing why Lucas (1976) contested their validity with respect to policy making.

1 Setting the stage

Aggregate demand and aggregate supply

As indicated by its name, the AD-AS model features an aggregate demand block of the economy and an aggregate supply block of the economy. We will further also partition the economy into a *real* side, which only features real goods, and a *nominal* side, which also features money.

In the aggregate demand block, the real side will be captured by the IS model, whereas the nominal side will be captured by the LM model. In turn, merging the two models will give rise to an aggregate demand structure that exhibits a 'two-way causality' between aggregate demand and the real rate of interest. Specifically, the IS model will determine aggregate demand as a function of the real rate of interest, whereas the LM model will determine the equilibrium real rate of interest as a function of aggregate demand. When both sides are congruent with one another, the resulting equilibrium gives rise to aggregate demand as a decreasing function of the aggregate price level. The negative, causal relationship between the aggregate price level and aggregate demand is illustrated by the *aggregate demand curve* shown in Figure 4.

In the aggregate supply block, the effect of a rising nominal price level on aggregate output depends on the assumption regarding the behavior of nominal wages. Specifically, if nominal wages are fully flexible, a nominal increase in the price level will be perfectly inconsequential because nominal wages will simply adjust proportionately so as to clear the market for labor. However, if nominal wages are sticky, then nominal shocks can have real effects by way of increasing or decreasing cost of labor in real terms. We will explore both of these assumption, the former under the term 'neoclassical model' and the latter under the term 'Keynesian model'.

Keynesian theory vs. real business cycles

At its core, the Keynesian philosophy can thus be reduced to the idea that real variables are not invariant to nominal changes. In this spirit, the AD-AS model is Keynesian because the nominal aggregate price level has the potential to affect both aggregate demand — through the real rate of interest — as well as aggregate supply — if wages are sticky. Similarly, many contemporary macroeconomic models feature characteristics to facilitate real effects induced by nominal changes generate, which is why we typically refer to such models as *New-Keynesian*. The Keynesian view is contrasted by real business cycle theory (RBC), which emphasizes the classical dichotomy — the neutrality of money — typically attributed to David Hume.[1]

Using axes to represent causality

Throughout this note, to promote intuition, I will rely on a number of graphs, of which there will be two general types. The first, more common type will depict a quantity as a dependent variable (Figure 4). The second, less common type will depict a price as a dependent variable (Figure 11). As alluded, I will make it a point to always represent the variable that is being determined in the corresponding setting on the y-axis. In this context, a somewhat peculiar case occurs when we switch from examining supply and demand in their own right versus their joint role in determining a market's equilibrium price. For example, both aggregate demand and aggregate supply take the aggregate price level as given such that it will be intuitive to assign the aggregate price level to the x-axis as shown in Figure 4. However, once we integrate aggregate demand and aggregate supply in the same graph, the variable being determined is the equilibrium price that clears the market. Therefore, it will be intuitive to assign the aggregate price level to the y-axis as in Figure 11. But don't economists always graph prices on the y-axis? They do, which just goes to show that economists, too, can sometimes be unreasonable.[2]

[1] The contrast between those two opposing views is related to the freshwater vs. saltwater divide among academic economics following the Lucas Critique. While freshwater schools such as the University of Chicago, the University of Minnesota, and the University of Rochester were dominated by proponents of RBC, saltwater schools such as Harvard University, Princeton University, and Columbia University found great value in Keynesian thinking. Today, we can conclude that freshwater thinking has prevailed to the extent that virtually all contemporary macroeconomic theory is 'microfounded' (as demanded by Lucas (1976), more on this later), whereas saltwater thinking has prevailed to the extent that virtually all contemporary macroeconomic theory features nominal frictions in the spirit of Keynes.

[2] Notice that being unreasonable does not amount to being wrong. Therefore, I will only deduct points for misrepresentations of causality if I remember to ask for a 'causal' assignment of the axes, which I hopefully will.

2 Aggregate demand

IS model

The IS model as captured by equations (IS1)-(IS5) determines aggregate demand Y^D *as a function* f^{IS} of the real rate of interest r given some predetermined level of government expenditures G^o,

$$\text{Aggregate demand:} \quad Y^D = C^D + I^D + G^D \tag{IS1}$$

$$\text{Consumer demand:} \quad C^D = \alpha_0 + \alpha_1(Y^I - G^D) - \alpha_2 r \tag{IS2}$$

$$\text{Firm demand:} \quad I^D = \alpha_3 - \alpha_4 r \tag{IS3}$$

$$\text{Government demand:} \quad G^D = G^o \tag{IS4}$$

$$\text{Consumer income:} \quad Y^I = Y^D \tag{IS5}$$

where I have assumed that all relationships are linear and $\alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3)$, all greater than zero, are a set of corresponding parameters. In effect, the IS model assumes that consumer demand and firm investment are decreasing in the real rate of interest. Moreover, consumer demand is increasing in net household income, the latter of which is decreasing in government expenditures because they are financed through taxes. To gain some intuition, let us consider a concrete example by setting $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4) = (2, 0.8, 0.025, 2, 0.025)$ and $G^o = 5$,

$$\begin{aligned}
\text{Consumer demand:} \quad & C^D = 2 + 0.8(Y^D - G^D) - 0.025r \\
\text{Firm demand:} \quad & I^D = 2 - 0.025r \\
\text{Government demand:} \quad & G^D = 5 \\
\Rightarrow \text{Aggregate demand:} \quad & Y^D = 2 + 0.8(Y^D - 5) - 0.025r + 2 - 0.025r + 5
\end{aligned}$$

which we can solve for Y^D as a function of r,

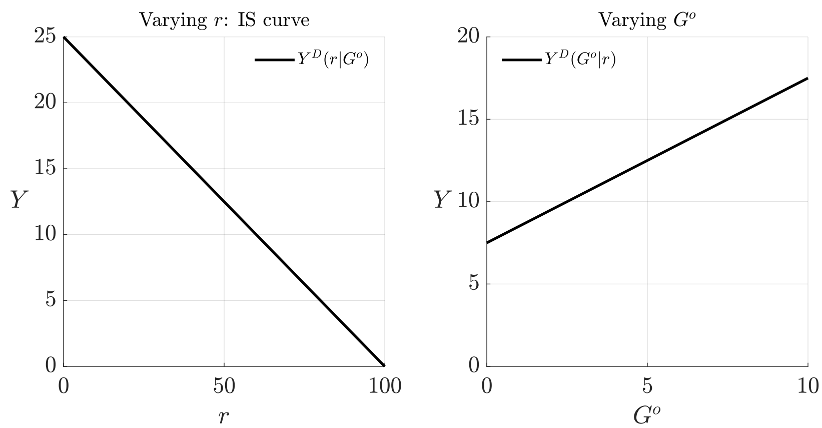
$$Y^D = 25 - 0.25r \tag{IS6}$$

such that aggregate demand is decreasing in the real rate of interest r for the chosen specification of α and G^o. More generally, without specifying α and G^o, it is possible to show that Y^D is given by,

$$Y^D = \underbrace{G^o + \overbrace{\frac{\alpha_0 + \alpha_3}{1-\alpha_1}}^{\alpha_5} - \overbrace{\frac{\alpha_2 + \alpha_4}{1-\alpha_1}}^{\alpha_6} r}_{\text{IS function } f^{IS}} \qquad (\text{IS}^\star)$$

such that, since $\alpha_2, \alpha_4 > 0$ and $\alpha_1 \in (0, 1)$, aggregate demand is decreasing in the real rate of interest for any permissible specification. In effect, the right-hand side of equation $\text{IS}^\star$ represents the IS function f^{IS} with inputs (G^o, r) and *parameterized* by α. To foster intuition, I now plot the IS function f^{IS} for the previously mentioned parameterization in Figure 1,

Figure 1. Different views of the IS function f^{IS}



Notes: Figure 1 illustrates the IS function f^{IS} as captured by equation $(\text{IS}^\star)$ for $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4) = (2, 0.8, 0.025, 2, 0.025)$. Notice that the view in Panel A corresponds to the relationship captured by equation (IS6). The primary insight from Figure 1 is that aggregate demand is decreasing in the real rate of interest r and increasing in government spending G^o. We further say that f^{IS} is linear because aggregate demand responds linearly to changes in r and G^o.

LM model

While the IS model determines aggregate demand as a function of the real rate of interest, the LM model determines the equilibrium real rate of interest as a function of aggregate demand. Here, the term equilibrium is used to emphasize the role of interest rates in equilibrating the supply of and demand for money. Specifically, the LM model consists of the following three equations,

$$\text{Money demand:} \quad M^D = P(\beta_0 + \beta_1 Y^D - \beta_2 r) \qquad (\text{LM1})$$

$$\text{Money supply:} \quad M^S = M^o \qquad (\text{LM2})$$

$$\text{Price level:} \quad P = P \qquad (\text{LM3})$$

where $\beta = (\beta_0, \beta_1, \beta_2)$ are parameters and the money stock M^o, and the aggregate price level P are predetermined. Importantly, money demand is increasing in aggregate demand, but decreasing in the real interest rate. To recover the relevant equilibrium condition, let us first construct excess demand money as follows,

$$\text{Excess demand:} \quad M^E = P(\beta_0 + \beta_1 Y^D - \beta_2 r) - M^o \tag{LM4}$$

Equilibrium in the market for money occurs when equation (LM4) is equal to zero with the real interest rate serving as the equilibrating price. In particular, we will assume that there exists some third party — the Walrasian auctioneer — who adjusts the real rate of interest right until the market for money clears. To gain some intuition, let us consider a concrete example by setting $(\beta_0, \beta_1, \beta_2) = (0, 2, 0.2)$, $M^o = 10$, and $P = 0.5$,

$$\begin{aligned} \text{Money demand:} \quad & M^D = P(2Y^D - 0.2r) \\ \text{Money supply:} \quad & M^S = 10 \\ \text{Price level:} \quad & P = 0.5 \end{aligned}$$

Equilibrium in the market for money is then ensured by an interest rate $r^\star$ such that excess demand $0.5(2Y^D - 0.2r^\star) = 10$, or,

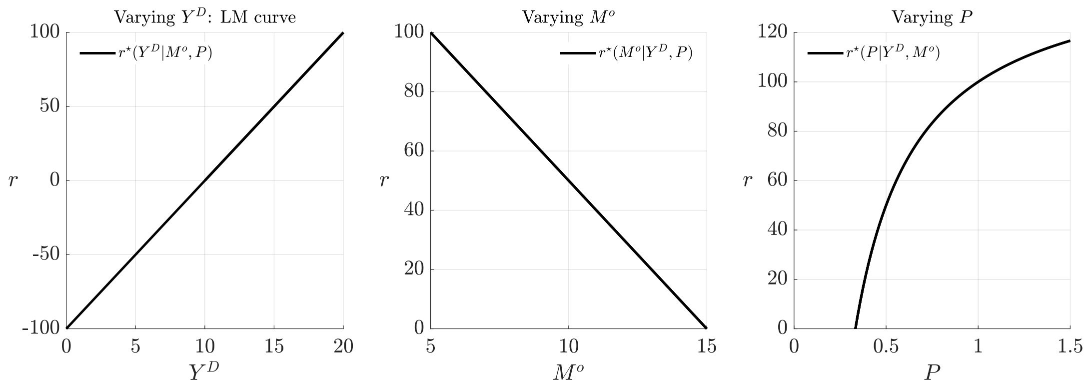
$$r^\star = 10Y^D - 100 \tag{LM5}$$

More generally, without specifying β, M^o, and P, it can be shown that the equilibrium interest rate $r^\star$ satisfies,

$$r^\star = \underbrace{\overbrace{\frac{\beta_0}{\beta_2}}^{\beta_3} + \overbrace{\frac{\beta_1}{\beta_2}}^{\beta_4} Y^D - \overbrace{\frac{1}{\beta_2}}^{\beta_5} \frac{M^o}{P}}_{\text{LM function } f^{LM}} \tag{LM*}$$

such that, since $\beta_0, \beta_1, \beta_2 > 0$, the equilibrium real rate of interest is increasing in Y^D, P, but decreasing in M^o for any permissible specification. In effect, the right-hand side of equation (LM*) represents the LM function f^{LM} with inputs (Y^D, M^o, P) and parameterized by β. To foster intuition, I now plot the LM function f^{LM} for a particular parameterization in Figure 2,

Figure 2. Different views of the LM function f^{LM}

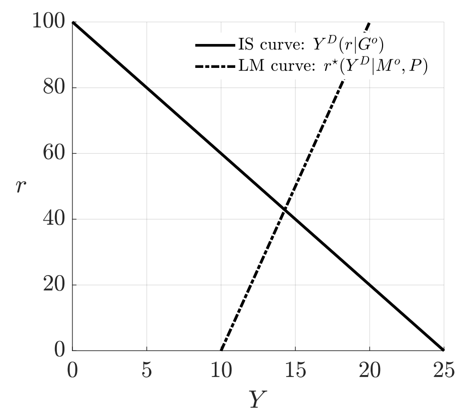


Notes: Figure 2 illustrates the LM function f^{LM} as captured by equation (LM*) for $(\beta_0, \beta_1, \beta_2) = (0, 2, 0.2)$. Notice that the view in Panel A corresponds to the relationship captured by equation (LM5). Panel A further also foreshadows a fundamental problem faced by the AD-AS model, namely that finding a parameterization that generates reasonable values for all variables is difficult at best. Nevertheless, the primary insight from Figure 1 is that the equilibrium real rate of interest is increasing in (real) aggregate demand Y^D and the price level P, but decreasing in the money stock M^o. We further say that f^{LM} is linear in Y^D and M^o because the equilibrium real rate of interest $r^\star$ responds linearly to such changes.

Merging IS and LM

Thus far, we have derived two functions. First, we had the IS function which determines aggregate demand as a function of the real rate of interest and government spending G^o. Second, we derived the LM function which determines the equilibrium real rate of interest as a function of aggregate demand, the existing money stock, and the aggregate price level. Whenever these two functions are congruent with one another, aggregate demand is in equilibrium in the sense that the market for money clears. Such a situation is illustrated in Figure 3, in which I have graphically plotted the IS curve against the LM curve.

Figure 3. Merging IS and LM



Notes: Figure 3 overlays the LM curve from Figure 2A with IS curve from Figure 1A. At their intersection, aggregate demand and the real interest rate are in equilibrium in the sense that the market for money clears.

To find the intersection in Figure 3 analytically, let us plug in (LM5) into (IS6),

$$\begin{aligned} Y^{D\star} &= 25 - 0.25r^\star \\ &= 25 - 0.25(10Y^{D\star} - 100) \\ &= \frac{100}{7} \end{aligned}$$

which implies $r^\star = \frac{300}{7}$. Money market equilibrium $(Y^{D\star}, r^\star)$ is thus given by $(\frac{100}{7}, \frac{300}{7})$, which we specifically derived under the assumption that $G^o = 5$ and M^o. We thus be interested to know how this equilibrium changes in response to changes in these fiscal and monetary policy variables, also known as the aggregate demand function.

Aggregate demand

To find an expression of aggregate demand as a function of M^o and G^o, we will need the functions f^{IS} and f^{LM} as captured by (IS⋆) and (LM⋆),

$$Y^D = \overbrace{G^o + \alpha_5 - \alpha_6 r}^{\text{IS function } f^{IS}} \tag{IS*}$$

$$r^\star = \underbrace{\beta_3 + \beta_4 Y^D - \beta_5 \frac{M^o}{P}}_{\text{LM function } f^{LM}} \tag{LM*}$$

where I have defined $\alpha_5 = \frac{\alpha_0+\alpha_3}{1-\alpha_1}$, $\alpha_6 = \frac{\alpha_2+\alpha_4}{1-\alpha_1}$, $\beta_3 = \frac{\beta_0}{\beta_2}$, $\beta_4 = \frac{\beta_1}{\beta_2}$, and $\beta_5 = \frac{1}{\beta_2}$ for ease of exposition. To find equilibrium aggregate demand $Y^{D\star}$, let us then plug f^{LM} into f^{IS} (or the other way around),

$$Y^{D\star} = \alpha_5 - \alpha_6 \left[\beta_3 + \beta_4 Y^{D\star} - \beta_5 \frac{M^o}{P}\right] + G^o$$

which finally yields,

$$Y^{D\star} = \overbrace{\gamma_0 + \gamma_1 \frac{M^o}{P} + \gamma_2 G^o}^{\text{AD function } f^{AD}} \tag{AD*}$$

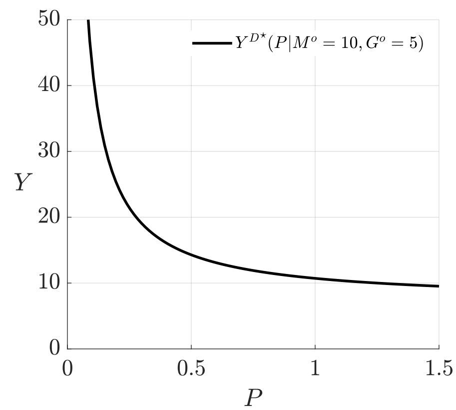
with the right-hand side of equation (AD⋆) representing the aggregate demand function f^{AD} with inputs (G^o, M^o, P) and parameterized by $\gamma_0 = \frac{\alpha_5-\alpha_6\beta_3}{1+\alpha_6\beta_4}$, $\gamma_1 = \frac{\alpha_6\beta_5}{1+\alpha_6\beta_4}$, and $\gamma_2 = \frac{1}{1+\alpha_6\beta_4}$. Since $\gamma_1, \gamma_2 > 0$, we can infer from (AD⋆) that equilibrium aggregate demand $Y^{D\star}$ is increasing in G^o

and M^o, but decreasing in P for any permissible specification,

$$\frac{\partial f^{AD}}{\partial G^o} > 0, \quad \frac{\partial f^{AD}}{\partial M^o} > 0, \quad \overbrace{\frac{\partial f^{AD}}{\partial P} < 0}^{\text{AD curve}} \tag{AD**}$$

It is the last property that gives the *aggregate demand curve*, which plots aggregate demand as a function of the aggregate price level its distinct downward sloping form as shown in Figure 4.

Figure 4. The aggregate demand curve

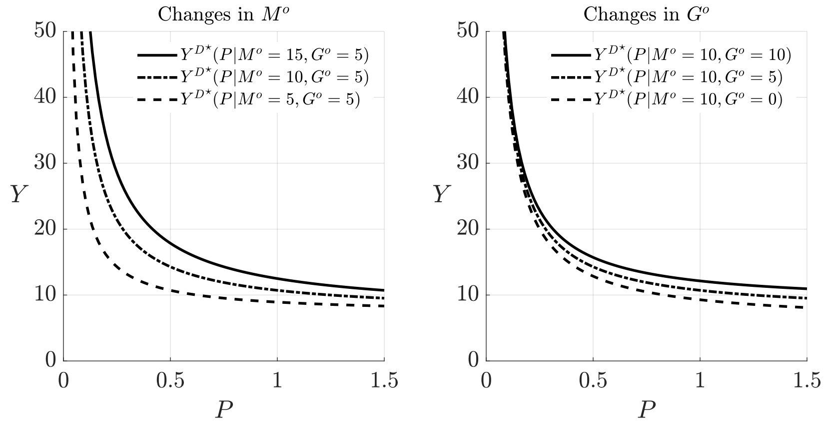


Notes: The canonical aggregate demand curve as illustrated in Figure 4 depicts the aggregate demand function f^{AD} as captured by (AD*) by plotting it for various levels of P while holding $G^o = 5$ and $M^o = 10$ fixed.

While it may be intuitive that demand is decreasing in a price, it is worth noting that the depicted curve depicts the real effect of a nominal change. In a frictionless world, one would not expect to see such effects. In particular, it should not matter whether some good costs USD100 or USD1000 as long as everyone's nominal balances (aka bank accounts) are scaled accordingly. The reason why aggregate demand is downward sloping in the IS-LM model is that a rising *nominal* price level induces upward pressure on the equilibrium *real* rate of interest via (LM*). Less controversially, the change in the real rate then causes households and firms to demand less goods via (IS2) and (IS3).

Since fiscal and monetary variables G^o and M^o are held fixed in Figure 4, it is natural to wonder how aggregate demand in response to exogenous changes in these variables. By (AD**), we know that aggregate demand is increasing in both G^o and M^o, but how can we best illustrate this? For this, we principally have two options. Usually, the effects of fiscal and monetary policy on aggregate demand are illustrated by continuing to plot $Y^{D\star}$ as a function of P as shown in Figure 5.

Figure 5. Shifts in the AD curve



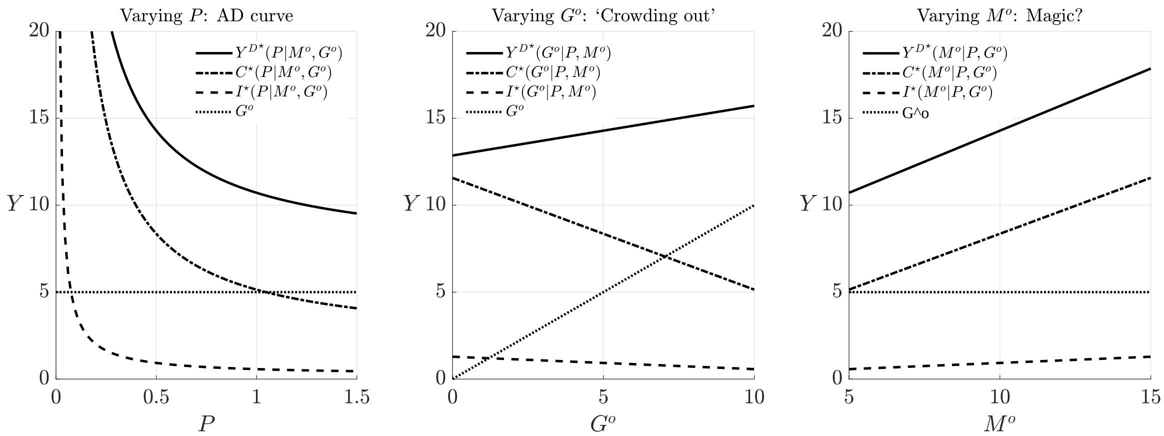
Notes: Figure 5 depicts shifts in the aggregate demand curve induced by exogenous variation in the fiscal and monetary variables G^o and M^o. By (AD**), we know that aggregate demand is increasing in both G^o and M^o such that the AD curve shifts up as these variables increase.

Since neither M^o nor G^o feature on the x-axis in either graph, the corresponding exogenous change manifests itself in the form of a shift of the aggregate demand curve. Examining the effects of exogenous changes in G^o and M^o on aggregate demand via the aggregate demand curve is intuitive, and it will be particularly helpful when adding aggregate supply into the mix, but let us, for now, explore the aggregate demand function f^{AD} from another angle. Specifically, rather than only plotting it against P, we can principally also plot it for varying levels of M^o and G^o themselves. This showing the (partial) effects of policy on aggregate demand more transparently, this approach also entails the benefit that we can decompose the effects of fiscal and monetary policy on output into consumption, investment, and government spending. Such an exercise is carried out in Figure 6

While Panels B and C of Figure 6 are quite illustrative in capturing the effects of monetary and fiscal policy on aggregate demand, they are often described as *partial equilibrium* because they hold the aggregate price level fixed. When the price level is allowed to adjust in response to fiscal and monetary policy in *general equilibrium*, such effects are virtually always mitigated (and sometimes even reversed). The difference between partial equilibrium effects and general equilibrium effects will be more clear once we merge aggregate demand with aggregate supply.[3]

[3] It will also be discussed in more detail at a later time.

Figure 6. Three different views of the aggregate demand function f^{AD}



Notes: Figure 6 illustrates the aggregate demand function as captured by (AD*) more broadly by plotting it against all of its three inputs (P, M^o, G^o). All three Panels show that investment accounts for a smaller share of aggregate demand than does consumption. Panel B illustrates that while increased government spending indeed boosts aggregate demand, it does so at the cost of 'crowding out' private consumption and investment. Panel C illustrates that increasing the stock of money induces a proportional increase in consumption and investment.

On a technical note, notice that comparing Figure 5 with Figure 6 helps us understand whether an exogenous change results in a shift along a curve or in a shift of the curve itself. Specifically, to make this distinction, we only need to check whether the variable that has undergone a change is represented on the x-axis of the graph in question (or y-axis if the graph's architect unreasonably insists on assigning the independent variable to the y-axis). If the changing variable is represented on the x-axis, then we are looking at a shift along the curve as shown in all three Panels of Figure 6. Conversely, if the perturbed variable is not represented on the x-axis, then the depicted curve will shift as shown in both Panels of Figure 5. For example, an exogenous increase in the money stock M^o can be represented as a shift of the aggregate demand *curve* by plotting $Y^{D\star}$ as a function of P (Figure 5A), or it can be represented as a shift along the aggregate demand *function* by plotting $Y^{D\star}$ as a function of M^o (Figure 6C).

3 Aggregate supply

Neoclassical supply

The aggregate supply curve depicts the equilibrium level of real output $Y^{S\star}$ as a function of the nominal price level P for a given technology z^o and capital stock K^o.

$$\text{Production:} \quad Y^S = z^o[\bar{N}]^\alpha[K^o]^{1-\alpha} \tag{NCAS1}$$

$$\text{Labor demand:} \quad N^D = \delta_0 A^o + \delta_1 K^o - \delta_2 w \tag{NCAS2}$$

$$\text{Labor supply:} \quad N^S = \kappa w \tag{NCAS3}$$

$$\text{Effective labor:} \quad \bar{N} = \min\{N^S, N^D\} \tag{NCAS4}$$

where the Walrasian auctioneer adjusts the real wage $w \equiv W/P$ so as to clear the market for labor. In such a market clearing equilibrium, the corresponding equilibrium wage $w^\star$ satisfies,

$$\underbrace{\kappa w^\star}_{N^S} = \underbrace{\delta_0 z^o + \delta_1 K^o - \delta_2 w^\star}_{N^D}$$

which implies,

$$w^\star = \underbrace{\frac{\delta_0}{\kappa + \delta_2}}_{\delta_3} z^0 + \underbrace{\frac{\delta_1}{\kappa + \delta_2}}_{\delta_4} K^o \tag{NCAS5}$$

with corresponding levels of equilibrium effective labor and output as follows,

$$N^\star = \kappa\delta_3 z^o + \kappa\delta_4 K^o \tag{NCAS6}$$

$$Y^{S\star} = \underbrace{z^o\left[\kappa\delta_3 z^o + \kappa\delta_4 K^o\right]^\alpha \left[K^o\right]^{1-\alpha}}_{\text{Neoclassical AS function } f^{AS}} \tag{NCAS$\star$}$$

Therefore, neoclassical aggregate supply only depends on technology z^o and capital K^o, but it is invariant to the aggregate price level P. First and foremost, this is because the Walrasian auctioneer is free to choose any real wage w to clear the labor market. We will consider an alternative, Keynesian specification of the economy shortly. In any case, let us now merge aggregate demand with aggregate supply to determine the aggregate price level $P^\star$ which clears the market for goods.

Neoclassical AD-AS

To determine the neoclassical AD-AS equilibrium, let us collect the AS and AD functions,

$$Y^{D\star} = \gamma_0 + \gamma_1 \frac{M^o}{P} + \gamma_2 G^o \tag{AD*}$$

$$Y^{S\star} = z^o \left[\kappa\delta_3 z^o + \kappa\delta_4 K^o\right]^{\alpha} \left[K^o\right]^{1-\alpha} \tag{NCAS*}$$

where aggregate demand is decreasing in P, whereas aggregate supply is invariant to P. Once again, we appeal to the Walrasian auctioneer to equate supply and demand by adjusting the aggregate price level right until $Y^{S\star} = Y^{D\star}$. Letting $P^\star$ denote the corresponding equilibrium price level, we have,

$$\underbrace{z^o \left[\kappa\delta_3 z^o + \kappa\delta_4 K^o\right]^{\alpha} \left[K^o\right]^{1-\alpha}}_{AS} = \underbrace{\gamma_0 + \gamma_1 \frac{M^o}{P^\star} + \gamma_2 G^o}_{AD}$$

which implies,

$$P^\star = \frac{\gamma_1 M^o}{z^o \left[\kappa\delta_3 z^o + \kappa\delta_4 K^o\right]^{\alpha} \left[K^o\right]^{1-\alpha} - \gamma_0 - \gamma_2 G^o} \tag{ADAS1}$$

$$Y^\star = z^o \left[\kappa\delta_3 z^o + \kappa\delta_4 K^o\right]^{\alpha} \left[K^o\right]^{1-\alpha} \tag{ADAS2}$$

$$N^\star = \kappa\delta_3 z^o + \kappa\delta_4 K^o \tag{ADAS3}$$

where $Y^\star$ denotes the equilibrium amount of output. Looking at (ADAS2) and (ADAS3), it is evident that in the neoclassical model, equilibrium labor and output are entirely determined by aggregate supply with no role for aggregate demand. This is unsurprising given the fact that aggregate supply is invariant to changes in P. For example, if P doubles, the Walrasian auctioneer will simply double the nominal wage W to keep the real wage constant, thereby ensuring that the labor market continues to clear via the market clearing condition (NCAS5).

The fact that equilibrium output is entirely determined by aggregate supply $Y^{S\star}$ without any role for aggregate demand $Y^{D\star}$ implies that any exogenous change that only affects aggregate demand, specifically (G^o, M^o), cannot affect equilibrium output $Y^\star$ or equilibrium labor $N^\star$. But how can this be? The reason why monetary policy and fiscal policy have no such effects is that all of their are absorbed by the aggregate price level. Specifically, while increased government spending

G^o or expansive monetary policy M^o do boost the demand for goods and services at any given price level P, such surges in demand will simply cause the Walrasian auctioneer to raise prices in order to clear the market at the only possible level of output, namely $Y^{S\star}$. Conversely, contractionary fiscal or monetary policy will cause prices to fall, once again with entirely inconsequential effects on equilibrium output. In effect, the main point made by the neoclassical AD-AS model is then captured by the following four partial derivatives,

$$\frac{\partial Y^\star}{\partial M^o} = 0 \qquad \frac{\partial N^\star}{\partial M^o} = 0 \qquad \frac{\partial P^\star}{\partial M^o} > 0$$
$$\frac{\partial Y^\star}{\partial G^o} = 0 \qquad \frac{\partial N^\star}{\partial G^o} = 0 \qquad \frac{\partial P^\star}{\partial G^o} > 0$$

As alluded to before, the reason why both fiscal and monetary policy are inconsequential in the neoclassical model is that any change in the aggregate price level is entirely absorbed by the Walrasian auctioneer presiding over the labor market. In his legendary book "The General Theory of Employment, Interest, and Money", Keynes hypothesized that labor markets may be subject to frictions that prevent real wages from clearing the market for labor. We will now discuss an alternative specification of aggregate supply, in which wages are 'sticky' and aggregate demand does have real effects.

Keynesian supply

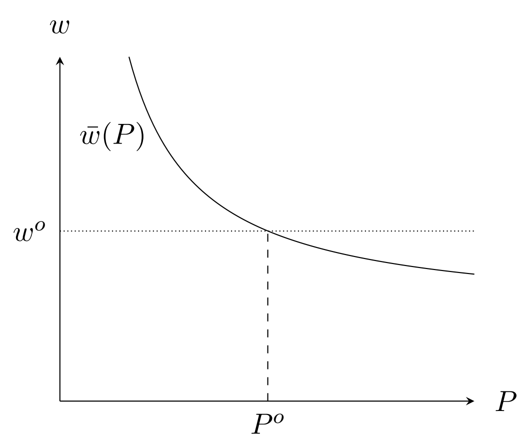
To understand the role of sticky wages, it is best to think of the economy as evolving in two stages. First, the economy is in equilibrium for an original level of the exogenous variables (z^o, K^o, G^o, M^o). At this stage, households and firms agree to labor terms (N^o, W^o) given the prevailing aggregate price level P^o. Here, the nominal wage is calculated as $W^o = w^\star P^o$, where $w^\star$ clears the market. In the second stage, one or more of the exogenous variables change, which causes a shift in aggregate supply according to (AS*), aggregate demand (AD*), or both. In turn, the Walrasian auctioneer reacts by starting to adjust the price level P from the original benchmark P^o. As P deviates from P, the ultimate equilibrium of the economy will depend on how flexible the nominal wage W is.

Before delving into the implications of sticky wages, I first define the process of how wages fail to fully adjust,

$$\bar{w} = (1-\lambda)w^\star + \lambda \frac{W^o}{P} \qquad \text{(NKAS1)}$$

where $\lambda \in (0,1)$ measures stickiness and the effective real wage $\bar{w}$ is equal to the labor market clearing wage $w^\star$ if $P = P$ as was the case in the first stage. In turn, as P starts deviating from P, real wages do not full adjust. Specifically, if the aggregate price level rises, real wages fall because they are partially still tied to the original nominal wage W^o. Conversely, if the aggregate price level falls, real wages rise for the same reason.

Figure 7. Effect of a price change on the real wage

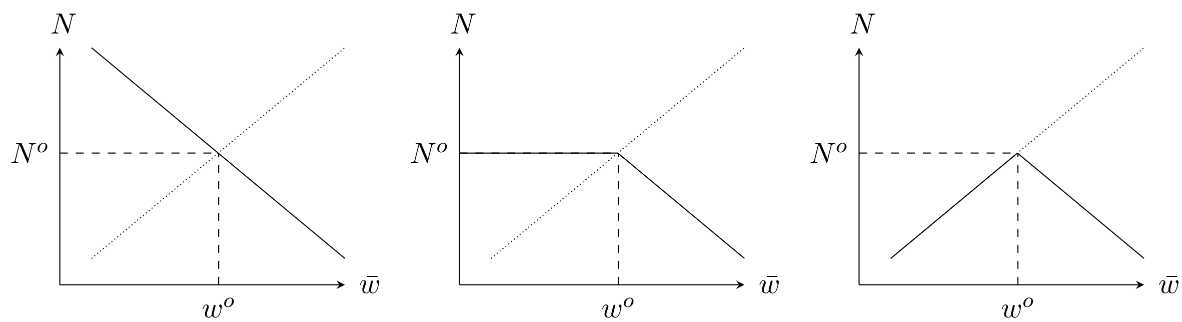


Notes: Figure 7 illustrates the effects of varying aggregate price levels on the effective real wage under a regime of sticky wages. Specifically, it is shown that the real wage is decreasing in the aggregate price level as the nominal wages W fail to fully adjust.

Now, to determine the equilibrium effects of (NKAS1), we have to make an assumption as to the structure of the labor contract agreed upon by the household and the firm. For this, one could principally imagine all sorts of contracts. A handful of examples of such contracts are,

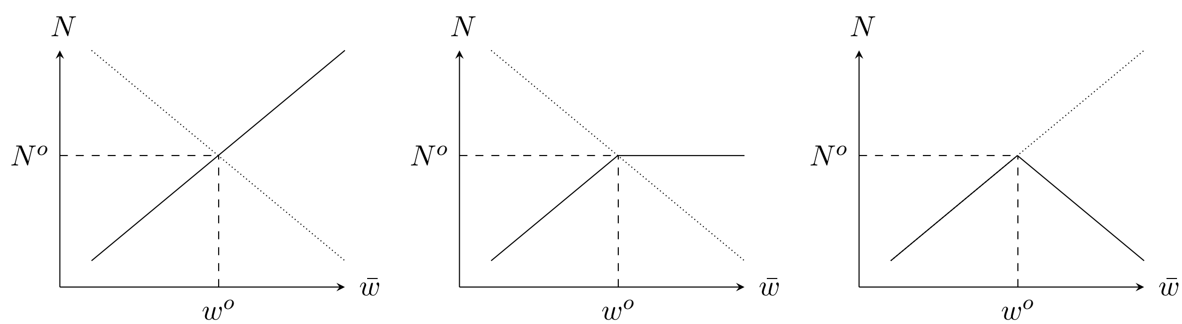
1. The household has committed to satisfying all labor demands (Figure 2A).
2. The household has committed to satisfying all labor demands up to N^o (Figure 2B).
3. The firm has committed to employing all labor supply (Figure 3A).
4. The firm has committed to employing all labor supply up to N^o (Figure 3B).
5. Neither households not firms have committed to satisfying anything (Figure 2C/3C).

Figure 8. Household commitment: a) full, b) limited, c) none



Notes: Figure 8 shows the varying effects of sticky wages on effective labor assuming different levels of commitment on the household side. Specifically, Panel A assumes full commitment, in which case effective labor is equal to labor demand. Panel B assumes partial commitment up to N^o, in which case effective labor is equal to N^o below w^o and equal to labor demand above w^o. Finally, Panel C assumes no commitment by either party, in which case effective labor is equal to labor supply below w^o and equal to labor demand above w^o.

Figure 9. Firm commitment: a) full, b) limited, c) none



Notes: Figure 9 shows the varying effects of sticky wages on effective labor assuming different levels of commitment on the firm side. Specifically, Panel A assumes full commitment, in which case effective labor is equal to labor supply. Panel B assumes partial commitment up to N^o, in which case effective labor is equal to labor supply below w^o and equal to N^o above w^o. Finally, Panel C assumes no commitment by either party, in which case effective labor is equal to labor supply below w^o and equal to labor demand above w^o.

Now, canonical Keynesian aggregate supply is built upon the first assumption, namely that households have committed to supply any amount of labor as demanded by the firm (Figure 8A). In this case, the labor demand equation (NCAS2) pins down effective labor with the effective wage $\bar{w}$ being set according to (NKAS1). Accordingly, Keynesian aggregate supply consists of the following equations, where I drop the star notation because, strictly speaking, the labor market does not clear,

$$\text{Effective wage:} \quad \bar{w} = (1-\lambda)w^\star + \lambda W^o/P \tag{NKAS1}$$

$$\text{Effective labor:} \quad \bar{N} = \delta_0 z^o + \delta_1 K^o - \delta_2 \bar{w} \tag{NKAS2}$$

$$\text{Production:} \quad Y^S = z^n [\bar{N}]^\alpha [K^n]^{1-\alpha} \tag{NKAS3}$$

where $W^o = Pw^\star$ is set in neoclassical fashion — according to (ADAS1) — given the original vector (z^o, K^o, M^o, G^o). Specifically, we can combine (ADAS1) with (NCAS5) to find W^o as follows,

$$W^o = \frac{\gamma_1 M^o[\delta_3 z^o + \delta_4 K^o]}{z^o\left[\kappa\beta_3 z^o + \kappa\delta_4 K^o\right]^\alpha \left[K^o\right]^{1-\alpha} - \gamma_0 - \gamma_2 G^o} \tag{NKAS4}$$

In turn, we can combine (NKAS1)-(NKAS4) to find Keynesian aggregate supply, which is no longer invariant to changes in the aggregate price level P. Although the algebra is somewhat cumbersome, we get,

$$Y^S = \underbrace{z^n\Bigg[\delta_0 z^n + \delta_1 K^n - \delta_2\Bigg((1-\lambda)w^\star + \lambda\Bigg[\frac{\gamma_1 M^o[\delta_3 z^o + \delta_4 K^o]}{\left[z^o\left[\kappa\beta_3 z^o + \kappa\delta_4 K^o\right]^\alpha \left[K^o\right]^{1-\alpha} - \gamma_0 - \gamma_2 G^o\right]P}\Bigg]\Bigg)\Bigg]^\alpha [K^n]^{1-\alpha}}_{\text{Keynesian AS } f_{NK}^{AS}} \tag{NKAS*}$$

Even though equation (NKAS*) looks rather complicated, its main point is straight forward, namely that Keynesian aggregate supply is increasing (upward sloping) in the aggregate price level P. To see this, we can either take the derivative of f_{NK}^{AS} with respect to P (no thanks), or we can sequentially follow the logic of (NKAS1)-(NKAS3): First, by (NKAS1), we know that the real wage is decreasing in P because nominal wages are sticky. In turn, as the real wage decreases, the firm demands more labor — by (NKAS2) — because labor has become cheaper. Finally, more labor turns into more output by (NKAS3). In effect, we have thus found that Keynesian aggregate supply Y^S is increasing in P because real wages fail to fully adjust and labor markets fail to clear,

$$\frac{\partial Y^S}{\partial P} > 0$$

Keynesian AD-AS

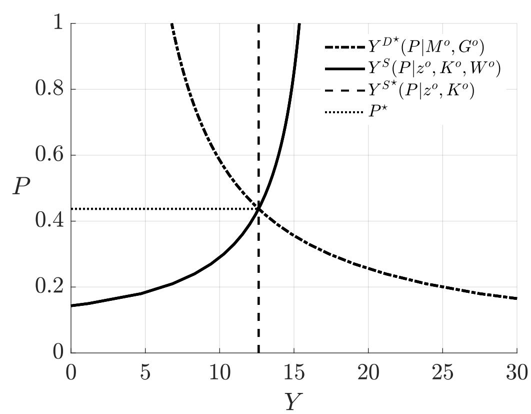
To determine equilibrium in the AD-AS model with Keynesian supply, let us collect the relevant AD and AS functions,

$$Y^{D\star} = \gamma_0 + \gamma_1 \frac{M^o}{P} + \gamma_2 G^o \tag{AD*}$$

$$Y^S = z^n \left[\delta_0 z^o + \delta_1 K^o - \delta_2 \left((1-\lambda) w^\star + \lambda \left[\frac{W^0}{P} \right] \right) \right]^\alpha [K^o]^{1-\alpha} \tag{NKAS*}$$

where W^o is determined according to (NKAS4) given (z^o, K^o, M^o, G^o) in the 'neoclassical first stage'. Crucially, aggregate demand is decreasing in P, whereas aggregate supply is increasing in P such that there must exist a unique price $P^\star$, at which the market for physical goods clears. In fact, given (z^o, K^o, M^o, G^o), we know that the market clearing price is equal to P, in which case the resulting equilibrium allocation $(P^\star, Y^\star)$ is equivalent to the one found in (ADAS1)-(ADAS3). This equivalence is shown in Figure 10, in which I have once flipped the axes because the equilibrium price $P^\star$ is now the object that is being determined (by the Walrasian auctioneer).

Figure 10. New-Keynesian vs. Neoclassical supply



Notes: Figure 10 depicts an aggregate demand curve as captured by (AD*) against neoclassical supply from (NCAS*) and Keynesian supply from (NKAS*). It can be seen that unless one of the exogenous variables (z^o, K^o, M^o, G^o) departs from its original value, the neoclassical equilibrium and the Keynesian equilibrium will be equivalent. As we shall see shortly, the same is not true if one of the exogenous variables (z^o, K^o, M^o, G^o) is perturbed.

Unfortunately, it is not possible to solve for $P^\star$ analytically — meaning with pen and paper — by setting supply Y^S from (AD*) equal to demand $Y^{D\star}$ from (NKAS*) as we did for neoclassical model. We will thus have to rely on graphical illustrations to understand what's going on.

4 Policy effects: Neoclassical vs. Keynesian

Before discussing the different effects of policy in our two different AD-AS models, it should be noted that even the neoclassical model (with neoclassical supply) is Keynesian to some degree in the sense that aggregate demand is not invariant to the aggregate price level. Keeping this in mind, I will continue to use the terms neoclassical AD-AS model and Keynesian AD-AS model as used throughout.

In the neoclassical model, monetary and fiscal policy are perfectly inconsequential in the sense that both equilibrium labor and equilibrium output are invariant to such policies. This is because the swings in aggregate demand caused by monetary or fiscal interventions are entirely absorbed by a corresponding increase or decrease in the equilibrium price level with real wages adjusting proportionately so as to clear the labor market. In contrast, Keynesian supply does not permit real wages to fully adjust, which leads to an upward sloping aggregate supply curve. As we shall see now, the upward sloping nature of aggregate supply will allow monetary policy to have real effects in that both labor and output are materially affected by such policies. The corresponding fiscal policy exercise will feature as a homework problem.

To gauge the effects of monetary policy in the Keynesian setup, I will focus on an *expansion* of the money stock from M^o to M^n. Prior to the expansion, the stage is set as depicted in Figure 10 such that $P = P$, $W = W^o$, and the labor market clears: $N^o = N^\star$. As the money stock rises from M^o to M^n, the newly relevant system of equations is given by,

$$Y^{D\star} = \gamma_0 + \gamma_1 \frac{M^n}{P} + \gamma_2 G^o \qquad \text{(AD}^\star\text{)}$$

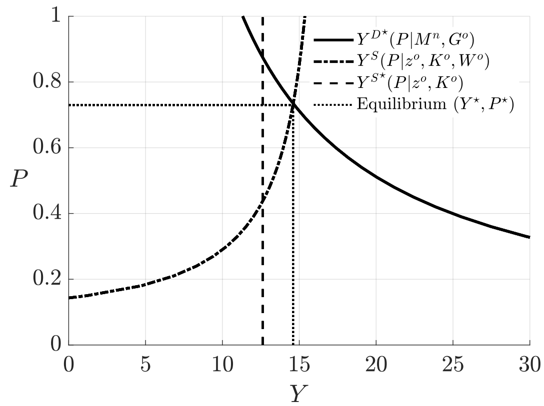
$$Y^S = z^n \left[\delta_0 z^o + \delta_1 K^o - \delta_2 \left((1-\lambda) w^\star + \lambda \left[\frac{W^0}{P} \right] \right) \right]^\alpha [K^o]^{1-\alpha} \qquad \text{(NKAS}^\star\text{)}$$

such that aggregate demand shifts to the right/up.[4] The resulting new equilibrium can be

[4]Whether the curve shifts right or up depends on our labeling of the axes. Specifically, if we were just examining

found by overlaying the unaffected aggregate supply curve with the new aggregate demand curve as shown in Figure 11.

Figure 11. A monetary expansion in the Keynesian AD-AS model



Notes: Figure 11 depicts the new aggregate demand curve implied by (AD*) following an increase in the money stock from M^o to M^n (with all other exogenous variables (z^o, K^o, G^o) held fixed). Since the Keynesian supply curve has a finite upward slope, the monetary expansion has a real effect in that equilibrium output $Y^\star$ has risen along with the rise in the aggregate price level.

Assume that the Walrasian auctioneer is happy to equilibrate aggregate demand with aggregate supply, the main point of Figure 11 is to illustrate that equilibrium output $Y^\star$ responds positively to a monetary expansion. We have thus successfully constructed a model, in which monetary policy has real effects. To critically assess the validity of our model along this dimension, let us retrace the underlying chain of causality step-by-step.

1. Initially, the rise in the money stock causes excess supply of money in equation (LM4). As always, the Walrasian auctioneer responds to a market in excess supply by lowering the price of the good, which in this case is the real rate of interest.

2. The auctioneer adjusts the interest rate right until excess demand for money is zero, in which case money demand must be equal to the newly changed supply. Since new supply is larger than the old supply, the new real rate must be lower than the previous rate by (LM1).

3. In response to the decrease real rate of interest, both households and firms increase their demand for physical goods as dictated by equations (IS2) and (IS3), thus shifting the AD curve to the right.

4. Since monetary policy does not affect aggregate supply in this model, the AS curve remains unchanged.

aggregate demand by itself, we would plot P on the x-axis, in which case the AD curve would shift up. In the context of AD-AS, however, we plot P on the y-axis because the equilibrium price $P^\star$ is the object that is being determined. In this context, the original upward shift turns into a rightward shift.

5 The Lucas Critique or the elephant in the room

Since macroeconomists do not typically have the chance to conduct counterfactual policy experiments, they often resort to constructing macroeconomic theories which allow them to do just that, albeit just hypothetically. A macroeconomic model's primary quality of interest is thus whether it serves as a valid instrument for counterfactual policy evaluation. To construct policy counterfactuals, it is of utmost importance that the model's quantities which are assumed to be fixed/policy-invariant are in fact fixed/policy-invariant. If this is not so, then a model's policy counterfactuals are dubious at best.

In the AD-AS context, notice that virtually none of our models' parameters ($\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_0, \beta_1, \beta_2, \sigma, \delta_0, \delta_1, \delta_2, \kappa, \lambda$) represent *deep parameters* in the sense that they allow for an intuitive, economic interpretation at a microeconomic level. However, the ability to lend an intuitive interpretation to a parameter is typically viewed as a prerequisite for the assertion that it is in fact policy-invariant. In fact, I would argue that only α_1, σ, and potentially λ allow for such an interpretation with the underlying assumptions that the savings rate, the marginal products of labor and capital, and wage stickiness are policy-invariant (all of which are arguably debatable). All other parameters do not allow for such an interpretation, which is why we call such parameters *reduced form*. At best, reduced form parameters represent a combination of some other, underlying deep parameters, in which case the model's policy counterfactuals are valid. If this is not so, then reduced form parameters represent other quantities which may appear stable, but which in actuality vary with policy. In effect, imposing reduced form parameters as model primitives is problematic because we can never be know if they are in fact policy-invariant or not. In particular, even if some estimated reduced form parameters work well in terms of fitting past data, they may fail spectacularly under a new policy regime (such as an increase in M^o or G^o).

In a nutshell, the above discussion summarizes the famous critique laid out by Lucas (1976), which effectively retired all models akin to the AD-AS models discussed herein. At the same time, the Lucas Critique marked the birth of a new type of model popularized by Kydland and Prescott (1977, 1982), which — in its most recent iteration — is called Heterogenous Agent New Keynesian Dynamic Stochastic General Equilibrium, or HANK DSGE for short. As you may be able to guess, HANK DSGE is way beyond the scope of this course, and so we will limit ourselves to a 'simple', yet 'microfounded' general equilibrium model which passes the Lucas' critique.

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