Rollover Crises in Calvo's Model of Sovereign Default (And How to Prevent Them)

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Abstract

In its canonical interpretation, Calvo (1988) describes a sovereign debt crisis in which deteriorating financing costs and rising debt levels feed back into each other in a self-fulfilling manner. By re-examining the original Calvo model through a decision-theoretic lens, we find that it also serves as a natural device to study crises in which the auction fails altogether (i.e. rollover crises). By considering solution concepts other than Nash equilibrium, our analysis not only provides a novel, non-self-fulfilling narrative of sovereign default, but it also inspires two simple levers to prevent such default.

Keywords: Sovereign default, epistemic game theory, Nash equilibrium, weak dominance

JEL codes: D80, C70, G01, H63

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1 Introduction

Since the seminal contribution by Calvo (1988), sovereign default is routinely interpreted as a real-world manifestation of multiple theoretical equilibria. Indeed, such multiplicity is appealing in that it provides an explanation for both crisis and non-crisis episodes within a unified economic framework. At the same time, it also leaves undetermined, or *indeterminate*, when or under which circumstances a crisis will in fact occur. In this paper, we illuminate and, ultimately, resolve this indeterminacy by re-examining the original Calvo theory through a decision-theoretic lens. Specifically, we compare the strategic environments induced by various epistemic states — i.e. investors' knowledge/assumptions about each other — thus forging a more nuanced understanding of the precise circumstances under which default can and/or will occur.

To motivate both crisis and non-crisis episodes within a unified framework, Calvo (1988) examines his theory through the lens of (Nash) equilibrium. That is, each investor is assumed to submit a bid that represents a best response to all other bids. While canonical, this modeling choice is epistemically delicate in that it implicitly presumes that all agents, among other things, either know each others' final choice and/or each others' beliefs (see Aumann and Brandenburger, 1995). Although potentially applicable in certain stylized circumstances, this assumption is almost surely too strong to describe an ordinary sovereign debt auction. Therefore, so long as informational barriers preclude investors from correctly anticipating each others' bids, Nash equilibrium does *not* represent an epistemically appropriate solution concept to study Calvo's auction. In turn, the main contribution of this paper lies in the derivation of an alternate solution concept which *does* suitably reflect an ordinary sovereign debt auction. This delineation is not only conceptually warranted, but it is also economically consequential in terms of its empirical predictions (see Section 2) and in terms of its implications for policy (see Section 3).

Various depths of weak dominance

To suitably capture the epistemic environment surrounding an ordinary sovereign debt auction[1], we consider various levels of weak dominance. To this end, we start with the basic premise that each investor is both rational and cautious, which allows for an elimination of one round of weakly dominated strategies (1EWDS). This is helpful in that it narrows down the types of bids

[1] See Aguiar and Amador (2013) for a relatively recent survey of the literature on sovereign debt.

that investors submit, but it notably does not rule out rollover crises as rationality and caution are insufficient to imply that investors submit a bid in the first place.[2] In our benchmark specification, rollover crises *can* be ruled out, however, if rationality and caution are mutually assumed (2EWDS).[3] In this case, investors successfully narrow down the types of bids that others submit and, thus, correctly infer that it is in their best interest to submit a bid themselves. Finally, a unique solution — Calvo's 'good' no-default equilibrium — emerges once a second layer of mutual assumption is added (i.e. everyone assumes that everyone assumes that everyone is rational and cautious — 3EWDS).[4]

While conceptually appealing, the fact that that three rounds of elimination of weakly dominated strategies (3EWDS) yield a unique prediction in Calvo's theory does not in and of itself render 3EWDS a contextually appropriate solution concept.[5] Instead, just like Nash equilibrium, 3EWDS is contextually appropriate if and only if its epistemic underpinnings — rationality, caution, and second-order mutual assumption thereof — represent an empirically accurate description of the particular strategic environment at hand.[6]

In selecting among the various aforementioned depths of weak dominance, we view second-order mutual assumption of rationality and caution (i.e. 3EWDS) as too strong an epistemic requirement to offer a sufficiently broad applicability. In turn, the key to determining which solution concept — 1EWDS or 2EWDS — is most appropriate lies in the following question: Aside from acting rationally/cautiously themselves, do investors assume that everyone else does so as well? From the point of view of internal consistency, the latter assumption would, of course, be warranted, but it is worth noting that committing to it exposes investors to non-trivial economic risks. Indeed, even if just a single investor were to submit an irrational bid, erroneously proceeding under the

[2]So long as investors believe that others might not be rational/cautious, they cannot rule out that the ultimately transacted interest rate will be undesirably high, in which case they prefer not submitting a bid at all. Thus, if a sufficient number of investors harbor the same fears, the auction fails altogether: a rollover crisis.

[3]In our benchmark specification, the deadweight cost of taxation is assumed to be sufficiently flat for bondholders to prefer higher yields (to lower yields) up to a certain upper bound. Conversely, if the deadweight cost were steep, bondholders would monotonically, and somewhat paradoxically, prefer lower yields to higher yields. In turn, in the latter case, rollover crises cannot be ruled out by *any* level of weak dominance, but policy still can (see Appendix A.2).

[4]The reader might wonder why 3EWDS rules out the 'bad' default equilibrium. Since bidding in accordance with the 'bad' equilibrium is weakly dominated, no investor who is both rational and cautious would ever choose to do so (see Section 2.3). Indeed, this equilibrium has been described as "fragile" and/or "unstable" (see Ayres et al., 2018).

[5]In fact, sovereign default is one of the few strands of the literature in which uniqueness may be perceived as a shortcoming rather than as a desirable theoretical property.

[6]This insight serves as an illustration of a broader point, namely that solution concepts ought to be chosen with reference to the epistemic state which they represent, not based on the conceptual appeal of the predictions that they imply (see Campbell and Mäder, 2023).

assumption that everyone is rational can lead to potentially substantial losses. Following this logic, we thus proceed by studying Calvo's auction through both 1EWDS and 2EWDS.

Policy implications

Differentiating between 1EWDS or 2EWDS is not only epistemically delicate, but it is also economically consequential. With rollover crises being ruled out by 2EWDS, but not 1EWDS, selecting between the two solution concepts constitutes an economically significant modeling choice (see Table 1). Thus, to err on the side of caution and not rule out rollover crises by assumption, we focus on 1EWDS as part of our counterfactual policy evaluation. In this context, the main question we seek to address is whether it is possible, by altering in some fashion the structure of the auction itself, to rule out rollover crises *even if* investors are unwilling to assume that everyone else is both rational and cautious, i.e. under 1EWDS.

Given the unusual source of rollover crises in our reading of Calvo's model — investors' fears of undesirably high interest rates — two interesting opportunities present themselves from the point of view of policy.[7] First, an obvious and simple way to alleviate such fears is to publicly announce an 'exclusion yield', i.e. an explicit cap on the range of acceptable bids, prior to the auction (see Section 3.1). Second, since specifying such an upper bound is both actuarily and politically delicate, the issuer might consider to solicit bids in the form of an interval instead. That is, rather than collecting minimum acceptable rates only, the issuing sovereign might invite investors to specify a maximum acceptable rate as well (see Section 3.2).

Under both of the referenced, relatively minor adaptations — with an exclusion yield and/or interval bidding — investors can safely participate in the auction even if they are unwilling to assume that others are rational and cautious. In this case, unlike under a traditional auction format (see Table 1), rationality and caution *are* in and of themselves sufficient to rule out rollover crises (see Tables 2 and 3).[8] This yields the main policy implication of our analysis, that is, so long as investors are rational and cautious themselves, but unwilling to assume that others are as well,

[7]Under ordinary circumstances, investors prefer higher interest rates to lower interest rates. However, if an asset's intrinsic value depends on the transacted rate itself, this preference may not prevail. For instance, in the Calvo model, there exists an interest rate threshold, beyond which the sovereign bond becomes undesirable because the government will repudiate. Thus, so long as investors cannot rule out that the ultimately transacted rate will fall in this undesirable range, not submitting a bid constitutes a best response to a permissible belief (see Figure 1).

[8]In turn, 2EWDS unsurprisingly implies the 'good' no-default Nash outcome. Thus, either adaptation — exclusion yields or interval bidding — effectively reduces the epistemic requirements to obtain the same set of predictions as under the original specification by one layer of mutual assumption.

imposing an exclusion yield and/or soliciting bids in the form of an interval can serve as effective measures to prevent rollover crises.

Related literature. By re-examining Calvo's seminal theory through a decision-theoretic lens, the present paper lies at the intersection of two strands of the literature: epistemic game theory and sovereign default.

A key objective of epistemic game theory is to explore and explicate the decision-theoretic assumptions that underlie existing game-theoretic solution concepts.[9] For example, in two groundbreaking contributions, Bernheim (1984) and Pearce (1984) characterize the strategic implications of common knowledge of rationality; calling the resulting solution concept *rationalizability*.[10] Further seminal contributions include the epistemic characterizations of Nash equilibrium (Aumann and Brandenburger, 1995), correlated equilibrium (Aumann, 1987), perfect and proper equilibrium (Blume, Brandenburger, and Dekel, 1991), and, more recently, weak dominance (Brandenburger, Friedenberg, and Keisler, 2008; see also Yang, 2015; Lee, 2016; Dekel et al., 2016; Catonini and DeVito, 2021; Keisler and Lee, 2023). Collectively, the main insight of the referenced papers is that each solution concept reflects a distinct epistemic state, only (or at most) one of which accurately describes any particular application of interest. That is, from an epistemic perspective, solution concepts are mutually exclusive. It is precisely this insight that we leverage in our re-examination of the sovereign debt auction described by Calvo (1988).

Despite its significant practical implications, epistemic game theory has received little attention from applied economic theory. Notable exceptions include first-price auctions (see Battigalli and Siniscalchi, 2003; Cho, 2005) and games featuring 'reputation' (see Watson, 1993; Battigalli and Watson, 1997). In a recent, thematically related example, Mäder (2024) argues that the strategic environment surrounding bank runs — depositors responding to each other's observed actions — is most appropriately captured by the solution concept of best-response dynamics (rather than Nash equilibrium or global games). Indeed, with strategic considerations playing a central role in many financial crisis frameworks, the financial crisis literature — including but not limited to sovereign default — serves as a natural pool for potential future applications of epistemic game theory.

[9]See Dekel and Siniscalchi (2015) for an overview.

[10]If players are allowed to correlate their actions, rationalizability is equivalent to iterated elimination of strictly dominated strategies (IESDS). If play must satisfy stochastic independence, rationalizability is stronger than IESDS.

For our decision-theoretic study of sovereign default, we concentrate on the auction by Calvo (1988) for three reasons. First, the contemporary literature has for all intents and purposes reached a consensus that sovereign default represents a real-world manifestation of multiple theoretical equilibria, a proposition dating back to Calvo (1988). In fact, after proving that equilibrium in Eaton and Gersovitz (1981) is unique, Auclert and Rognlie (2016) interpret as follows:

> "Our objective is not to deny that sovereign debt markets can be prone to self-fulfilling crises [...]. Instead, we hope that our results may help sharpen the literature's understanding of the assumptions that are needed for such multiple equilibria to exist."

Second, Calvo's theory is highly adaptable in that it can be extended to account for both 'rollover crises' and 'slow moving crises'.[11,12] Third, the Calvo model's tractable two-period nature allows for a comprehensive epistemic inspection without requiring any further simplifying assumptions.

Aside from an economically significant policy implication — that exclusion yields and interval bidding prevent rollover crises — our re-examination of Calvo (1988) additionally provides a novel account of how a rollover crisis might unfold. Specifically, under the canonical narrative of sunspot-driven strategic coordination between multiple equilibria, rollover crises arise because investors correctly anticipate that others are unwilling to pay a positive price and, thus, optimally elect to forgo the auction themselves (see Cole and Kehoe, 2000; Bocola and Dovis, 2019; and Conesa and Kehoe, 2022). In contrast, in our case, rollover crises arise because investors *do not know* if and/or what others are bidding. In distinguishing between these two types of rollover crises, we neither wish to assert that our narrative is universally applicable, nor that it is superior.[13] Instead, our analysis simply suggests that rollover crises need not, as stipulated by Nash equilibrium, be the product of investors correctly anticipating each others' behavior. In other words, rollover crises need not be self-fulfilling.

[11] In a recent extension of Calvo's auction, Lorenzoni and Werning (2019) distinguish between the two referenced types of crises. A *rollover crisis* — as initially described by Alesina et al. (1990) and explored in more depth by Cole and Kehoe (2000), Bocola and Dovis (2019), and Conesa and Kehoe (2022) — occurs when a sovereign fails to roll over its expiring debt, namely because investors are unwilling to pay a positive price for the newly issued bond. A *slow moving crisis*, on the other hand, occurs when deteriorating financing costs and increasing debt levels slowly feed back into each other prior to default.

[12] To the reader familiar with this literature, the fact that Calvo's auction can generate rollover crises may come as a surprise. Indeed, through the canonical lens of Nash equilibrium, this is not the case. However, once we relax the implicit assumption that investors correctly anticipate each others' bids, they very well might optimally decide not to submit a bid at all (see Section 2). In turn, if a sufficient number of investors choose to forego the auction, the latter fails altogether: a rollover crisis.

[13] Ultimately, which of the two narratives is contextually appropriate depends, as we have emphasized, on the specific epistemic environment of the relevant auction at hand.

2 An indeterminate sovereign debt auction

In this section, we re-examine the auction described in Calvo (1988) through a decision-theoretic lens. For this, we start by collecting the epistemic assumptions that are required to induce equilibrium and then contrast said epistemic state with a series of alternative states characterized by rationality, caution, and various levels of mutual assumption thereof.

2.1 Setup

Consider a set of n investors all of whom possess a single unit of real balances to be invested in either physical capital at a predetermined return R or in a government bond b at a yield R_b to be determined in a sealed-bid auction.[14] Specifically, to purchase shares of the bond, investor i may submit an interest rate bid $R_{bi} \in \mathbb{R}_{>0}$ at or above which they commit to buy, or they may choose to forgo the auction by choosing $R_{bi} = 0$.[15] Following the auction, bids are filled sequentially (at a uniform rate R_b), starting with the lowest positive bid, until the entire issue is sold. Bids that are filled fully ($f_i = 1$) are called *successful bids*, bids that are filled partially ($f_i \in (0, 1)$) are called *marginal bids*, and bids that are left unfilled ($f_i = 0$) are called *unsuccessful bids*. Ultimately, the transacted rate R_b is set equal to the lowest unsuccessful bid or, if the set of unsuccessful bids is empty, to the highest submitted bid,[16]

$$R_b = \begin{cases} \min_{i \in I}\{R_{bi} | f_i = 0, R_{bi} > 0\} & \text{if} \quad \{i | f_i = 0\} \neq \emptyset \\ \max_{i \in I}\{R_{bi}\} & \text{if} \quad \{i | f_i = 0\} = \emptyset \end{cases} \tag{1}$$

While the first case of (1) captures an auction that is 'well oversubscribed' (because there exists at least one unsuccessful bid), the second case represents three separate instances in which no submitted bid is unsuccessful: a (well or barely) oversubscribed auction featuring multiple marginal bidders or a barely oversubscribed auction featuring a single marginal bidder.[17] In turn,

[14] We assume that total investor wealth significantly exceeds the government's external financing needs, i.e. $n \gg b$.

[15] Investors who forgo the auction are assumed to invest all their balances in physical capital.

[16] See Appendix B for the alternate case when R_b is set equal to the highest successful bid (i.e. marginal pricing).

[17] An oversubscribed auction is said to be *well oversubscribed* if it features at least one unsuccessful bid or multiple marginal bids, whereby no singular bidder can raise their bid while maintaining their marginal status. Conversely, an oversubscribed auction is said to be *barely oversubscribed* if it features (i) no unsuccessful bids and (ii) either a single marginal bid or multiple marginal bids, whereby at least one bidder could raise their bid while maintaining their marginal status (because all others are collectively 'small' relative to the unsubscribed portion of the bond). Thus, since winning bids can influence the ultimately transacted rate if the auction is barely oversubscribed, strategizing *can* pay off in such a scenario.

if the auction is undersubscribed, we assume that it is cancelled with all household wealth being diverted to physical capital.

Finally, to complete the game, we are left to define utility. For this, suppose that the capital return R is risk free, whereas investments into the government bond might yield a return less than R_b, namely if the government decides to repudiate. In the latter case, the bonds' actual return is given by $(1-\theta)R_b$, where $\theta \in [0,1]$ reflects the share of debt that the government repudiates. In turn, following Calvo (1988), we assume that investor utility is increasing in consumption with the latter satisfying,

$$c_i = y + f_i[1-\theta]R_b + (1-f_i)R - [x + z(x)]/n$$

where y denotes a uniform income earned post-auction and z captures the deadweight cost associated with aggregate taxation x (see Appendix A).[18]

Now, if θ were predetermined (and known by all investors), optimal play would be straightforward.[19] However, the main idea put forth in Calvo (1988) — and embraced by a large literature on sovereign default thereafter — is that we should expect the credit risk associated with sovereign debt to be increasing, at least weakly, in its own interest rate.[20] Specifically, Calvo (1988) considers a model in which the government's (optimal) repudiation share θ satisfies,

$$\theta = \underbrace{[bR_b + g - h_x(R_b)]/[(1-\alpha)bR_b]}_{h_\theta(R_b|b,g,\alpha)}$$

where h_x reflects the government's (optimal) choice of taxation as depicted in Figure 1, g denotes government spending, and $\alpha \in (0,1)$ is the pro-rata cost of repudiation. In effect, a key conceptual issue that arises in Calvo's model is that when investors submit their bids, their 'reservation rate' $R/[1-\theta]$ — as reflected by the indifference condition in Figure 1 — is not yet known because it depends on the outcome of the auction itself (via h_θ). Thus, so long as the outcome of the auction is not known ex ante, investors must resort to forming beliefs. In this context, to narrow down the set of permissible investor beliefs, we start with the basic assumption that the government's operations are commonly known.

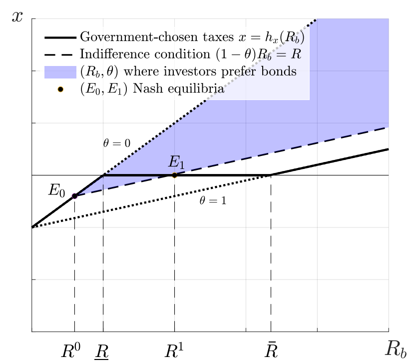
[18]To allow for outcomes in which not all investors hold the same portfolio ex post, we require subscripts.

[19]Assuming competitive bidding (i.e. an oversubscribed auction), winning bids cannot influence the ultimately transacted rate R_b. Thus, each investor would submit their predetermined reservation rate (i.e. the lowest rate they are willing to accept) such that we would have to have $R_b = R_{bi} = R/(1-\theta)$ for each i.

[20]In Calvo (1988), since default is certain when $\theta > 0$, θ does not technically reflect credit *risk*.

(A) *Assuming* the government's payoffs and rationality are common knowledge, each investor understands $x = h_x(R_b)$ and $\theta = h_\theta(R_b)$ (as shown in Figure 1) to be common knowledge.

Figure 1. Taxes, repudiation, and equilibrium in Calvo (1988)



Notes: Figure 1 reproduces, in a slightly augmented fashion, Figure 2 in Calvo (1988). It depicts the government's choice of taxation x as well as the resulting repudiation share θ as a function of the auction-implied bond yield R_b. As such, Figure 1 contains three main insights. First, equilibrium is indeterminate. Second, investors strictly prefer bonds when $R_b \in (R^0, R^1)$, and capital when $R_b > R^1$. Third, as the yield rises above R^0, either taxes x or the repudiation share θ increase, thus influencing investor welfare (see Appendix A).

2.2 Correlated/Nash equilibrium

Following the common practice of motivating strategic coordination between multiple equilibria via extrinsic uncertainty (i.e. sunspots), we start by interpreting each equilibrium depicted in Figure 1 as a manifestation of correlated equilibrium à la Aumann (1987).

(B) *Assuming investors can distinguish between two extrinsic states of the world* (ξ_0, ξ_1) *where each investor anticipates everyone to play* R^0 *in state* ξ_0*, and* R^1 *in state* ξ_1, then no investor, assuming they act rationally in both states, has an incentive to deviate from said play. In turn, while the resulting global play forms a correlated equilibrium, local play — since each investor's play is mutually known in both states — is Nash.

Although perhaps applicable in a few extraordinary instances, the main premise underlying Assumption (B) — that each investor correctly anticipates everyone else's actions — is almost surely too strong to describe an ordinary sovereign debt auction. Moreover, it is entirely unclear how and/or why investors might coordinate onto E_1. Indeed, since investors strictly prefer to hold bonds if and only if $R_b \in (R^0, R^1)$, bidding R^1 is weakly dominated by any bid in said interval. Thus, so long as investors are cautious in that they wish to account, even if just in a lexicographic sense, for the possibility that the ultimately transacted rate might fall into the interior of $[R^0, R^1]$, rationality dictates that they would never choose to play R^1.

2.3 Rationality, caution, and mutual assumption thereof

To formally account for the fact that bidding R^1 is weakly dominated, this subsection explores a series of epistemic states featuring investor caution. To this end, we proceed by assuming that in the interval $[R^0, \underline{R}]$, investors not only prefer to hold bonds, but that bondholder welfare is actually increasing in R_b.[21]

(C) *Assuming each investor is rational and cautious*, no bids below R^0 or in excess of $\underline{R}$ are submitted.[22]

Interestingly, although Assumption (C) significantly narrows down the types of bids that are submitted, it is insufficient to rule out a rollover crisis, namely because forgoing the auction is strictly preferred to any bid in $[R^0, \underline{R}]$ whenever $R_b > R^1$. In turn, to rule out this possibility, we require an additional epistemic assumption,

(D) *Assuming payoffs are mutually known and each investor assumes* (C), they correctly infer that $R_b \leq \underline{R} < R^1$ so long as the auction is successful.[23] Thus, given their cautious nature, it is in their best interest to submit a bid such that we have $R_{bi} \in [R^0, \underline{R}]$ for each i.

Finally, to obtain a unique prediction, one might consider adding another layer of mutual assumption,

(E) *Assuming each investor assumes* (D), they correctly infer that the issue will be well oversubscribed such that no winning bid can influence the transacted rate R_b.[24] Thus, given their cautious nature, $R_{bi} = R^0$ is the unique remaining cautiously rational strategy for each i.

In summary, in the sovereign debt auction studied by Calvo (1988), rationality, caution, and second-order assumption thereof (alongside second-order mutual knowledge of payoffs) are sufficient to imply that each investor, after correctly deducing that everyone else is submitting a bid in $[R^0, \underline{R}]$, optimally bids R^0. However, if at least one of the above listed epistemic conditions does not hold, other outcomes — including a rollover crisis — are conceivable (see Table 1).

[21] Whether this is true principally depends on whether the deadweight cost z is sufficiently flat (see Appendix A.1). However, even if this is not the case, the paper's main insight and policy implications continue to hold (see Appendix A.2).

[22] Since any bid in $(0, R^0)$ is weakly dominated by $R_{bi} = R^0$ and any bid in $(\underline{R}, \infty)$ is weakly dominated by $R_{bi} = \underline{R}$, any full-support LPS over opponents' strategies rules out such bids. However, so long as bondholder welfare is increasing in $[R^0, \underline{R}]$ and a winning bid can, at least in principle, influence R_b, bidding in $(R^0, \underline{R}]$ is *not* weakly dominated by $R_{bi} = R^0$. At the same time, $R_{bi} = R^0$ is *not* weakly dominated by any bids in $(R^0, \underline{R}]$, namely because there always exists an outcome in between, in which case $R_{bi} = R^0$ strictly beats such an 'interior bid'.

[23] This inference is correct to the extent that (C) actually holds. If it does not, falsely assuming that it does can lead to significant losses. Indeed, depending on the number of investors who choose to forgo the auction, even just one irrational bid may be sufficient to push the ultimately transacted rate above the critical threshold R^1.

[24] See Appendix B for the alternate case when R_b is set equal to the highest successful bid (i.e. marginal pricing).

Table 1. Various epistemic states and outcomes in Calvo's original model (1988)

Epistemic state	Solution concept	Outcome
Rationality, mutual know. of choice and/or common know. of beliefs	Nash equilibrium	$R_b = R_{bi} = R^0\ \forall i$ (no default eq.) or $R_b = R_{bi} = R^1\ \forall i$ (default equilibrium)
Rationality and caution	1EWDS	$R_b, R_{bi} \in [R^0, \underline{R}] \cup \{0\}\ \forall i$
Rationality, caution, and first-order mutual assumption thereof	2EWDS	$R_b, R_{bi} \in [R^0, \underline{R}]\ \forall i$
Rationality, caution, and second-order mutual assumption thereof	3EWDS	$R_b = R_{bi} = R^0\ \forall i$ (i.e. no-default eq.)

Table 1 compares various well-known solution concepts' underlying epistemic assumptions as well as their implied strategic outcomes in Calvo's auction. For example, if investors are rational and correctly anticipate each others' bids, the two canonical Nash outcomes obtain (E_0 and E_1 in Figure 1). Conversely, if bidding is driven by rationality, caution, and mutual assumption thereof, a variety of outcomes are conceivable. For example, 3EWDS uniquely implies the no-default equilibrium, whereas 1EWDS only rules out bids below R^0 and in excess of $\underline{R}$. Most importantly, 1EWDS does *not* rule out that investors forgo the auction altogether, thus paving the way for rollover crises.

3 Preventative measures

From a policy perspective, the key to preventing rollover crises in the Calvo model lies in the assurance of (cautious) investors that they can safely submit a bid without running the risk of receiving shares at a rate in excess of the critical threshold R^1 (see Figure 1). To this end, since the power to specify the auction's terms lies with the issuer, this section discusses two issuer-mandated policies to prevent rollover crises under 1EWDS.

3.1 Exclusion yields

A simple, but effective mechanism to prevent the ultimately transacted rate from exceeding the critical threshold R^1 is to ex ante cap the range of acceptable bids. That is, prior to the auction, the issuer might announce that they are only willing to accept bids equal to or below a maximum acceptable bid $R_b^{\max} = R^1$. In this case, investors need not worry about receiving shares of the bond at an undesirably high rate such that each epistemic state shown in Table 1 leads to a corresponding new set of conceivable outcomes.

(C') *Assuming each investor is rational and cautious*, everyone submits a bid $R_{bi} \in [R^0, \underline{R}]$. In particular, this is because investors know that their bid will not be filled in excess of R^1 such that not submitting a bid is weakly dominated. Moreover, since influencing the ultimately transacted rate is principally possible (i.e. if the auction is barely oversubscribed), bidding in excess of R^0 is both rational and cautious.

(D') *Assuming payoffs are mutually known and each investor assumes (C')*, they recognize that the auction will be well oversubscribed such that no winning bid can influence the ultimately transacted rate. Thus, $R_{bi} = R^0$ is the unique remaining cautiously rational strategy.

Thus, as shown in Table 2, ex ante capping the range of acceptable bids effectively reduces, by precisely one layer of mutual assumption, the epistemic requirements to obtain the same predictions shown in Table 1.

Table 2. Various epistemic states and outcomes in Calvo's model with $R_b^{\max} = R^1$

Epistemic state	Solution concept	Outcome
Rationality and caution	1EWDS	$R_b, R_{bi} \in [R^0, \underline{R}]\ \forall i$
Rationality, caution, and first-order mutual assumption thereof	2EWDS	$R_b{=}R_{bi}{=}R^0\ \forall i$ (i.e. no-default eq.)

Table 2 revisits two previously examined epistemic states (see Table 1) under the assumption that Calvo's auction is augmented with an exclusion yield of $R_b^{\max} = R^1$. In this instance, since investors need not worry that their bid will be filled at an undesirably high rate, rationality and caution are in and of themselves sufficient to rule out rollover crises. In turn, adding a single layer of mutual assumption yields the canonical no-default Nash outcome.

Interestingly, real-world practices that constrain the set of acceptable bids do in fact exist. Specifically, Italian sovereign bond auctions typically feature so-called "exclusion yields" which effectively amount to a reservation price for the auctioned bond.[25]

Although our theoretical analysis supports the use of exclusion yields, we anticipate three practical obstacles that may limit their effectiveness in preventing a rollover crisis. First, real-world investors may not share a common valuation of the bond such that a uniform critical threshold may not exist. Second, even if a uniform critical threshold does exist, it may be non-trivial for the issuer to elicit its precise value.[26] Finally, exclusion yields may be perceived as a drastic intervention into the market's ability to determine prices. As such, their imposition may be met with suspicion and, ultimately, drive certain investors away from the auction. In summary, since imposing an appropriate exclusion yield is both actuarily and politically delicate, we now consider an alternate policy, whereby investors' latitude is not constrained, but enhanced.

[25]While Italian exclusion yields are designed to protect the sovereign from "speculative behavior" (see Beetsma et al., 2020), we show that they can, albeit in a slightly different form (specified ex ante rather than as a function of the submitted bids), principally also serve to protect the sovereign from rollover crises.

[26]Indeed, in Italy sovereign bond auctions, exclusion yields are defined as a function of the received bids. In this form, exclusion yields would, of course, be ineffective in preventing rollover crises.

3.2 Interval bidding

A key premise that underlies point-based (i.e. 'lower-bound-only') bidding is that investors are willing to accept any rate above their submitted bid, namely because they strictly prefer higher to lower interest rates. While likely true in most circumstances, an exception to this premise may arise if an asset's intrinsic value depends on the transacted rate itself. For example, Stiglitz and Weiss (1981) study a model in which banks optimally ration credit because raising interest rates leads to a deterioration of the resulting loans' credit risk. Similarly, in Calvo (1988), there exists an interest rate threshold, beyond which the auctioned sovereign bond becomes undesirable because investors correctly anticipate that the government will repudiate. Thus, in both cases, lenders only prefer higher interest rates *up to a certain point.*[27] In turn, a natural mechanism to allow investors to express this non-monotonicity in preference is to solicit bids in the form of an interval.

Interval bidding

Rather than submitting a point-based bid, suppose now that investors are invited to submit bids in the form of an interval, i.e. $R_{bi} \in \mathcal{K}_{\geq 0}$, where $\mathcal{K}_{\geq 0}$ denotes the set of all convex sets satisfying $\{x \in \mathbb{R} | \underline{\mathcal{R}}_{bi} \leq x \leq \bar{\mathcal{R}}_{bi}, \bar{\mathcal{R}}_{bi} \in \mathbb{R}_{\geq 0} \cup \{\infty\} \geq \underline{\mathcal{R}}_{bi} \in \mathbb{R}_{\geq 0}\}$.[28]

With interval bidding of the described type, a question that naturally arises is how the ultimately transacted rate R_b will, or should, be determined. In this context, the first thing to note is that the decision on how to map bids into R_b lies, just like the decision on how to conduct the auction itself, with the issuer. Thus, to specify a favorable pricing mechanism from the point of view of the issuer, we start with the basic observation that issuers, unlike investors, *do* have monotonic preferences: lower interest rates are always preferred to higher interest rates. In effect, a seemingly natural way to proceed is to fill bids sequentially starting with the lowest positive lower bound.[29] Specifically, to minimize investors' incentive to strategize, the issuer might decide to set R_b equal to the lowest non-winning, positive lower bound or, if each submitted bid is at least filled partially, to the highest submitted lower bound.[30]

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[27] In our baseline specification (where the deadweight cost z is relatively flat), this point is $\underline{R}$. However, if z is sufficiently steep, investors actually monotonically prefer lower interest rates to higher interest rates (see Appendix A.1).

[28] We include positive infinity as a permissible upper bound so as to nest the traditional point-based ('lower-bound-only') system.

[29] With interval bidding, we assume that $\{\underline{\mathcal{R}}_{bi}, \bar{\mathcal{R}}_{bi}\} = \{0, 0\}$ is used to represent the choice of forgoing the auction.

[30] As before, without any further assumptions, the pricing mechanism captured by (2) does not rule out strategizing behavior entirely, namely because winning bids can, at least in principle, influence the ultimately transacted rate.

$$R_b = \begin{cases} \min_{i\in I}\{\underline{\mathcal{R}}_{bi} | f_i = 0, \underline{\mathcal{R}}_{bi} > 0\} & \text{if} \quad \{i | f_i = 0\} \neq \emptyset \\ \max_{i\in I}\{\underline{\mathcal{R}}_{bi}\} & \text{if} \quad \{i | f_i = 0\} = \emptyset \end{cases} \tag{2}$$

where f_i continues to denote the fraction of a submitted bid which gets filled. As under point-based bidding, the second case of (2) captures three separate instances in which there are no unsuccessful bids: a (well or barely) oversubscribed auction featuring multiple marginal bidders or a (barely) oversubscribed auction featuring a single marginal bidder.[31]

To examine the strategic implications of our newly proposed auction structure, let us now revisit the epistemic states (C) and (D),

(C") *Assuming each investor is rational and cautious*, everyone submits a bid with lower bound $\underline{\mathcal{R}}_{bi} \in [R^0, \underline{R}]$ and upper bound $\bar{\mathcal{R}}_{bi} = R^1$.[32]

As before, (C") significantly narrows down the types of bids that are submitted. Indeed, the above referenced set of conceivable bids closely mirrors the corresponding set under point-based bidding, albeit subject to two exceptions. First, rather than (implicitly) agreeing to buy at any rate in excess of the submitted rate, investors unsurprisingly take advantage of the ability to cap their bid at the critical threshold R^1. Second, since there is no risk associated with submitting a bid under (optimal) interval bidding, not submitting a bid is now weakly dominated. Thus, unlike under point-based bidding, (C") *does* rule out rollover crises under interval bidding.

In a final step, we reconsider the case in which rationality and caution are mutually assumed,

(D") *Assuming payoffs are mutually known and each investor assumes* (C"), they recognize that the auction will be well oversubscribed such that no winning bid can influence the ultimately transacted rate. Thus, each investor reveals their valuation truthfully, i.e. $\{\underline{\mathcal{R}}_{bi}, \bar{\mathcal{R}}_{bi}\} = \{R^0, R^1\}$, which, by (2), implies $R_b = R^0$.

In summary, both exclusion yields and interval bidding weaken the epistemic assumptions that are required (i) to rule out rollover crises and (ii) to obtain the 'good' no default equilibrium. Intuitively, this is because both policies reduce the epistemic requirements to ensure that investors

[31] We continue to assume that undersubscribed auctions are cancelled.

[32] Since an investor's chosen lower bound can, at least in principle, influence the ultimately transacted rate (i.e. when the auction is barely oversubscribed), no bid $\underline{\mathcal{R}}_{bi} \in [R^0, \underline{R}]$ is weakly dominated. In particular, this is because while raising $\underline{\mathcal{R}}_{bi}$ lowers the chances of getting filled, but it also bears the potential to increase one's payoff in a barely oversubscribed auction.

submit a bid in the first place: In a traditional auction, investors must (be willing to) assume that everyone else is rational and cautious, a requirement that is nullified by a suitably chosen exclusion yield and/or by the ability to specify an upper bound under interval bidding.

Table 3. Various epistemic states and outcomes in Calvo's model with interval bidding

Epistemic state	Sol. concept	Outcome
Rationality and caution	1EWDS	$\underline{\mathcal{R}}_{bi} \in [R^0, \underline{R}]$, $\bar{\mathcal{R}}_{bi} = R^1$ $\forall i$, $R_b \in [R^0, \underline{R}]$
Rationality, caution, and first-order mutual assumption thereof	2EWDS	$\{\underline{\mathcal{R}}_{bi}, \bar{\mathcal{R}}_{bi}\} = \{R^0, R^1\}$ $\forall i$, $R_b = R^0$

Table 3 revisits, once more, two previously examined epistemic states (see Tables 1 and 2), now assuming that investors are invited to submit bids in the form of an interval. In this instance, to avoid having their bid will be filled at an undesirably high rate, investors trivially choose $\bar{\mathcal{R}}_{bi} = R^1$. In turn, the resulting set of conceivable outcomes are analogous to the outcomes obtained under a suitably chosen exclusion yield (see Table 2).

4 Discussion

In this section, we collect the main policy implications of our analysis and discuss a conceptual point of interest: the unusual (i.e. not self-fulfilling) nature of rollover crises in Calvo's model (under 1EWDS).

On policy: Opportunities and challenges

We have proposed two separate policies — exclusion yields and interval bidding — to rule out rollover crises in Calvo's model of sovereign debt when investors are rational and cautious (i.e. 1EWDS). At their core, both policies operate identically, namely by eliminating investors' fear that their bid will be filled at an undesirably high interest rate. Exclusion yields eliminate such fears by introducing an issuer-imposed upper bound on the range of acceptable bids, whereas interval bidding achieves the same objective by allowing investors to specify their own, individual upper bounds. In turn, since both exclusion yields and interval bidding allow investors to safely submit bids (i.e. without running the risk of incurring a loss), they render forgoing the auction a weakly dominated strategy and, as such, are sufficient to rule out rollover crises (so long as investors are both cautious and rational).

While both of our policy proposals successfully eliminate rollover crises in theory, there are important practical challenges to consider. First, while exclusion yields constrain investors' latitude, interval bidding boosts it. Thus, if there are a large number of investors who are suspicious of issuer-imposed constraints, interval bidding may represent a more suitable option. Second, and perhaps even more importantly from a practical perspective, a key reason why both policies work in our theory is that investors value the auctioned asset identically. Thus, to the extent that this 'common value' assumption may not hold in practice, it is important that we consider the robustness of our results to an alternate, 'private value' specification.

The fact that investors may value a given sovereign bond differently presents, at least theoretically, a challenge for both of our proposed policies. Indeed, to be effective, a suitable exclusion yield must be set high enough to exceed a sufficient number of investors' minimum acceptable rate, but it must also be set low enough so as to not exceed the same set of investors' maximum acceptable rate. Analogously, allowing investors to submit their own range of acceptable rates entails the risk of obtaining bids that are non-overlapping.[33] However, to the extent that, in practice, said range is likely of the order of at least a few percent for each investor, we are hopeful that the theoretical challenge arising from differing valuations is precisely just that: theoretical. Nevertheless, prior to imposing an exclusion yield and/or soliciting bids in the form of an interval, a sovereign's office of debt management ought to be aware of such theoretical possibilities.

Rollover crises: self-fulfilling vs. not self-fulfilling

Aside from the policy implications described above, the main conceptual insight from our analysis is that rollover crises need not be self-fulfilling. Specifically, the reason why investors might hesitate to participate in Calvo's auction is not that they fear that others are forgoing the auction. Instead, the source of their hesitancy is that they cannot rule out that others *are* submitting bids, namely ones in excess of the critical threshold R^1 (see Figure 1). Indeed, if an investor anticipated a rollover crisis in Calvo's model, submitting a bid constitutes a best response.[34] Thus, rollover crises are not self-fulfilling.

The reason why rollover crises are typically viewed/modeled as self-fulfilling is purely method-

[33] In general, non-overlapping bids are non-trivial to resolve, but they do not present an issue so long as investors (i) share a common valuation of the item for sale and (ii) have an incentive to reveal said valuation truthfully. While the first condition — a common valuation — happens to be satisfied in Calvo's auction, the second can be ensured by way of proper auction design. Specifically, to incentivize investors to reveal their valuations truthfully, pricing must ensure that no winning bid can influence R_b. Indeed, if investors are aware that they cannot influence the ultimately transacted rate (as ensured by 2EWDS, see Table 3), they can at best maximize the likelihood that their bid will be filled at a desirable rate, which immediately implies $R_{bi} = \{\underline{\mathcal{R}}_{bi}, \bar{\mathcal{R}}_{bi}\} = \{R_b^0, R_b^1\}$ for each investor.

[34] In particular, this is because we assumed that undersubscribed auctions are cancelled.

ological. Indeed, by studying economic theory through the canonical lens of Nash equilibrium, we implicitly assume that each player correctly anticipates everyone else's behavior such that any resulting prediction, unique or indeterminate, must be self-fulfilling.[35] Thus, to obtain predictions that are not self-fulfilling, we must be willing to consider outcomes (and, thus, solution concepts) that are not Nash.

Although we find that rollover crises *need not* be self-fulfilling, we do not wish to suggest that they *cannot* be self-fulfilling. Nevertheless, prior to placing a bond in a volatile and/or otherwise uncertain market environment, it would be imprudent for a sovereign to only consider strategic scenarios in which investors successfully coordinate their expectations when, in fact, a rollover crisis might just as well arise from investors' inability to coordinate.

5 Conclusion

After re-examining Calvo's canonical model of sovereign default though a decision-theoretic lens, we conclude that, to accurately capture the strategic environment surrounding an ordinary sovereign debt auction, the solution concepts of 1EWDS or 2EWDS are more appropriate (depending on the network of authorized primary market bidders).[36] Under 1EWDS — i.e. if investors are rational and cautious themselves, but unwilling to assume that everyone else is as well — the Calvo model gives rise to (non-self-fulfilling) rollover crises which can be prevented by way of imposing a suitable exclusion yield and/or soliciting bids in the form of an interval.

[35]It may be counterintuitive to label a unique equilibrium as self-fulfilling, but Nash equilibrium, at least in its pure strategy form, is self-fulfilling by definition.

[36]As such, our analysis serves as a practical illustration of a key lesson from epistemic game theory, namely that solution concepts ought to be chosen with reference to the epistemic state which they represent, not based on the conceptual appeal of the predictions that they imply (see Campbell and Mäder, 2023).

A Alternate specification

Our baseline specification proceeds under the assumption that bondholder welfare is strictly increasing between R^0 and $\underline{R}$ (and strictly decreasing thereafter). In particular, this is because the marginal benefit of a rising bond yield is assumed to outweigh, at least locally, the marginal costs associated with increased taxation (inclusive deadweight costs). The aim of this section is twofold. First, Appendix A.1 explicates the condition that must hold for bondholder welfare to be increasing between R^0 and $\underline{R}$. In turn, Appendix A.2 discusses an alternate specification, whereby bondholder welfare is monotonically decreasing in R_b. Unsurprisingly, the strategic implications vary across the two specifications, but the main insight — that the Calvo model serves as a natural device to study rollover crises — and the main policy implication — that exclusion yields and interval bidding prevent rollover crises — continue to hold.

A.1 Welfare

To trace the welfare effects of an increase in R_b, we require a specification of utility. To this end, we assume that utility is strictly increasing in consumption and, following Calvo (1988), assume that consumption obeys the following equation,[37]

$$c_i = y + \overbrace{[1 - f_i]R}^{\text{capital}} + \overbrace{f_i(1-\theta)R_b}^{\text{bonds}} - \overbrace{[x + z(x)]/n}^{\text{taxes+deadweight cost}}$$

where y is income earned post-auction. To examine the effects of an increase in R_b, we distinguish between no repudiation ($\theta = 0$), partial repudiation ($\theta \in (0, 1)$), and full repudiation ($\theta = 1$).

Case 1: No repudiation

When $R_b \in [R^0, \underline{R}]$, we have $\theta = 0$ (i.e. no repudiation) and $\partial x/\partial R_b = b$ in which case the welfare effects of a rise in R_b are calculated as follows,

$$\frac{\partial c_i}{\partial R_b} = \overbrace{f_i}^{\text{yield}\uparrow} - \overbrace{b/n}^{\text{taxes}\uparrow} - \overbrace{z'(x)(b/n)}^{\text{deadweight cost}\uparrow}$$

[37] In the equilibrium framework considered by Calvo (1988), a representative investor holds b units of bonds and k units of capital. Conversely, to allow for heterogenous portfolios across n investors with one unit of endowment each, we rewrite the consumption equation in an analogous form.

where investors may or may not benefit from the increase depending on what fraction of their wealth is held in bonds. For example, the welfare of an investor who exclusively holds capital (i.e. $f_i = 0$) is clearly decreasing in R_b (because of increased taxation and the deadweight cost associated with taxation). Conversely, so long as $f_i > b/n$, the benefits associated with the bond's increased yield outweigh the corresponding direct costs associated with increased taxation. However, as captured by the last summand, total welfare must also account for the deadweight cost associated with increased taxation. For example, the maximal marginal benefit of increasing R_b is incurred by an investor who exclusively holds bonds (i.e. $f_i = 1$). In this case, we have,

$$\frac{\partial c_i}{\partial R_b} = 1 - (b/n)[1 + z'(x)]$$

such that, so long as the deadweight cost is sufficiently flat (i.e. $z'(x^\star) < n/b - 1$), the welfare of an investor who exclusively holds bonds is strictly increasing for any $R_b \in [R^0, \underline{R}]$. Conversely, if the deadweight cost is sufficiently steep (i.e. $z'(x^0) > n/b - 1$), the welfare of an investor who exclusively holds bonds is strictly decreasing for any $R_b \in [R^0, \underline{R}]$.

Case 2: Partial repudiation

When $R_b \in (\underline{R}, \bar{R})$, we have $\theta \in (0, 1)$ (i.e. partial repudiation) and $\partial x/\partial R_b = 0$ in which case the welfare effects of a rise in R_b are calculated as follows,

$$\frac{\partial c_i}{\partial R_b} = \overbrace{f_i}^{\text{yield} \uparrow} - \overbrace{f_i/(1-\alpha)}^{\text{repudiation} \uparrow}$$

Thus, so long as the deadweight cost satisfies $\alpha \in (0, 1)$, we must have that $\partial c_i/\partial R_b < 0$ for any $R_b \in (\underline{R}, \bar{R})$ irrespective of f_i.

Case 3: Full repudiation

Finally, when $R_b \geq \bar{R}$, we have $\theta = 1$ (i.e. full repudiation) and $\partial x/\partial R_b = \alpha b$ in which case the welfare effects of a rise in R_b are calculated as follows,

$$\frac{\partial c_i}{\partial R_b} = -\overbrace{\alpha b/n}^{\text{taxes} \uparrow} - \overbrace{z'(x)(\alpha b/n)}^{\text{deadweight cost} \uparrow}$$

which implies $\partial c_i/\partial R_b < 0$ for any $R_b \geq \bar{R}$ irrespective of f_i.

A.2 Monotonic bondholder welfare

Since investor welfare is strictly decreasing in R_b above $\underline{R}$ (irrespective of what fraction of wealth is held in bonds), bidding in excess of $\underline{R}$ is weakly dominated. In turn, if deadweight costs are sufficiently flat (i.e. $z'(x^\star) < n/b - 1$), an investor who (permissibly) believes that their whole bid will be filled might rationally bid up to $\underline{R}$. Conversely, if deadweight costs are sufficiently steep (i.e. $z'(x^0) > n/b - 1$), bidding in excess of R^0 is weakly dominated, namely because welfare is strictly decreasing in R_b above R^0 (irrespective of what fraction of wealth is held in bonds). This subsection examines the strategic implications of the latter case (i.e. when bondholder welfare is monotonic).

(C*) *Assuming each investor is rational and cautious*, no bids other than R^0 are submitted. However, rather than bidding R^0, investors may rationally opt to forgo the auction altogether.

To understand why cautious investors may rationally choose to forgo the auction, the first point to note is that investor welfare is weakly decreasing in R_b. Thus, even though investors strictly prefer bonds over capital whenever $R_b \in (R^0, R^1)$, they also prefer, perhaps surprisingly, lower bond yields to higher bond yields.[38] Following this logic, since bidding R^0 weakly dominates any bid in excess of R^0, an investor who is both rational and cautious will only submit a bid equal to R^0. In turn, whether such an investor decides to submit a bid at all depends on their beliefs regarding the bidding behavior by others. Specifically, if they deem it likely that $R_b \in [R^0, R^1]$, they would rather hold bonds and, thus, rationally choose to submit a bid (equal to R^0). Conversely, if they deem it likely that $R_b \in (R^1, \infty)$, they would rather hold capital and, thus, rationally choose to forgo the auction. In summary, while bidding R^0 weakly dominates any bid in excess of R^0, said strategy neither weakly dominates nor is weakly dominated by the choice of forgoing the auction.

Since cautious investors may rationally opt to forgo the auction, rationality and caution are insufficient, just like under the original specification, to rule out a rollover crisis. Indeed, if a sufficient number of investors worry that R_b might exceed R^1, the auction is undersubscribed and the sovereign fails to roll over its debt. In turn, since no cautiously rational investor submits a bid in excess of R^1, one might conjecture that adding a layer of mutual assumption (of rationality and caution) will be sufficient to rule out rollover crises. Unlike in our baseline specification, this is not the case,

[38] This is true irrespective of whether they hold bonds or capital, namely because taxes must be paid either way.

(D*) *Assuming payoffs are mutually known and each investor assumes* (C*), they correctly infer that no investor submits a bid other than R^0. However, in this case, capital and bonds yield the exact same (risk-free) return such that investors are perfectly indifferent between submitting a bid and not submitting a bid.[39]

Even though adding additional layers of mutual assumption (of rationality and caution) are insufficient to rule out rollover crises when bondholder welfare is monotonic, exclusion yields and interval bidding continue to be effective. Indeed, so long as investors need not fear that they will receive shares of the bond at a rate in excess of R^1, they have no incentive to forgo the auction and thus, so long as they are rational and cautious, submit a bid equal to R^0. That is, when bondholder welfare is monotonic and all investors are both rational and cautious, the only conceivable outcome is given by the good Nash equilibrium E^0.

[39] More precisely, so long as all wealth is diverted towards capital when the auction fails, investors are indifferent between bidding and forgoing the auction under their first lexicographic layer of beliefs (as captured by Assumption A3'). Moreover, as already established under Assumption A2', neither bidding nor forgoing the auction weakly dominates the other under their second lexicographic layer of beliefs.

B Marginal pricing

In this section, we examine the strategic implications of marginal pricing (while assuming that the deadweight costs are relatively flat, i.e. $z'(x^\star) < n/b - 1$). That is, what if, rather than setting the ultimately transacted rate R_b equal to the lowest unsuccessful bid, R_b is set to the highest successful bid instead? We find that marginal pricing precludes a unique prediction, namely because investors have an incentive to strategize even if the auction is well oversubscribed. To see this, consider first the canonical case of point-based bidding (without an exclusion yield),

$$R_b = \begin{cases} \max_{i \in I}\{R_{bi} | f_i > 0\} & \text{if} \quad \{i | f_i = 0\} \neq \emptyset \\ \max_{i \in I}\{R_{bi}\} & \text{if} \quad \{i | f_i = 0\} = \emptyset \end{cases} \tag{1'}$$

where $R_{bi} \in \mathbb{R}_{\geq 0}$ for each i. The key difference between our benchmark specification (1) and marginal pricing as captured by (1') is that, under marginal pricing, winning bids *can* influence the ultimately transacted rate even if the auction is well oversubscribed.[40] We thus have,

($C^\dagger$) *Assuming each investor is rational and cautious*, no bids below R^0 or in excess of $\underline{R}$ are submitted. However, not submitting a bid cannot be ruled out as it is strictly preferred to any bid $R_{bi} \in [R^0, \underline{R}]$ whenever $R_b > R^1$.[41]

($D^\dagger$) *Assuming payoffs are mutually known and each investor assumes* ($C^\dagger$), they correctly infer that as long as they submit a bid in $[R^0, \underline{R}]$ themselves, any resulting auction-implied rate R_b will lie in the same interval. Thus, it is in fact in their best interest to submit a bid and we have $R_{bi} \in [R^0, \underline{R}]\ \forall i$.

($E^\dagger$) *Assuming each investor assumes* ($D^\dagger$), they correctly infer that the issue will be well oversubscribed. However, since marginal bidders retain the ability to influence pricing even in the event of a well oversubscribed auction, the same solution, i.e. $R_{bi} \in [R^0, \underline{R}]\ \forall i$, prevails.

Finally, consider the same pricing mechanism with an exclusion yield of R^1.

($C^\ddagger$) *Assuming each investor is rational and cautious*, everyone submits $R_{bi} \in [R^0, \underline{R}]$.

($D^\ddagger$) *Assuming payoffs are mutually known and each investor assumes* ($C^\ddagger$), everyone recognizes that the auction will be well oversubscribed. However, since marginal bidders retain the ability to influence pricing even in the event of a well oversubscribed auction, the same solution, i.e. $R_{bi} \in [R^0, \underline{R}]\ \forall i$, prevails.

[40] For example, suppose a well oversubscribed auction features a single marginal bidder. Clearly, since (1') sets R_b equal to the marginal bidder's bid, the latter has the ability to influence R_b.

[41] Crucially, marginal pricing does not imply that investors are automatically protected from undesirably high rates, namely because others may be bidding in excess of R^1 (and pricing is uniform).

References

Aguiar, Mark and Manuel Amador. 2013. *Handbook of International Economics*, chap. Sovereign Debt. Elsevier, vol. 4 ed.

Alesina, Alberto, Alessandro Prati, and Guido Tabellini. 1990. *Public Debt Management: Theory And History*, chap. Public Confidence And Debt Management: A Model And A Case Study Of Italy. Cambridge University Press and CEPR.

Auclert, Adrien and Matthew Rognlie. 2016. "Unique Equilibrium in the Eaton-Gersovitz Model of Sovereign Debt." *Journal of Monetary Economics* 84:134–146.

Aumann, Robert J. 1987. "Correlated Equilibrium as an Expression of Bayesian Rationality." *Econometrica* 55 (1):1–18.

Aumann, Robert J. and Adam Brandenburger. 1995. "Epistemic Conditions for Nash Equilibrium." *Econometrica* 63 (5):1161–1180.

Ayres, João, Gaston Navarro, Juan Pablo Nicolini, and Pedro Teles. 2018. "Sovereign default: The role of expectations." *Journal of Economic Theory* 175:803–812.

Battigalli, Pierpaolo and Marciano Siniscalchi. 2003. "Rationalizable bidding in first-price auctions." *Games and Economic Behavior* 45:38–72.

Battigalli, Pierpaolo and Joel Watson. 1997. "On "Reputation" Refinements with Heterogeneous Beliefs." *Econometrica* 65 (2):369–374.

Beetsma, Roel, Massimo Guiliodori, Jesper Hanson, and Frank de Jong. 2020. "Determinants of the bid-to-cover ratio in Eurozone sovereign debt auctions." *Journal of Empirical Finance* 58:96–120.

Bernheim, Douglas B. 1984. "Rationalizable Strategic Behavior." *Econometrica* 52 (4):1007–1028.

Blume, Lawrence, Adam Brandenburger, and Eddie Dekel. 1991. "Lexicographic Probabilities and Equilibrium Refinements." *Econometrica* 59 (1):81–98.

Bocola, Luigi and Alessandro Dovis. 2019. "Self-fulfilling Debt Crises: A Quantitative Analysis." *American Economic Review* 109 (12):4343–4377.

Brandenburger, Adam, Amanda Friedenberg, and Terri Kneeland. 2008. "Admissibility in Games." *Econometrica* 76 (2):307–352.

Calvo, Guillermo A. 1988. "Servicing the Public Debt: The Role of Expectations." *American Economic Review* 78 (4):647–661.

Campbell, Jason and Nicolas Mäder. 2023. "Deriving Predictions from Economic Theory: Solution Concepts vs. Epistemic Conditions." *Working Paper* .

Catonini, Emiliano and Nicodemo De Vito. 2021. "Cautious Belief and Iterated Admissibility." *Working Paper* .

Cho, In-Koo. 2005. "Monotonicity and Rationalizability in a Large First Price Auction." *Review of Economic Studies* 72:1031–1055.

Cole, Harold L. and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crises." *The Review of Economics Studies* 67 (1):91–116.

Conesa, Juan Carlos and Timothy J. Kehoe. 2022. "Roll-Over Crises in Sovereign Debt Markets." *Working Paper* .

Dekel, Eddie, Amanda Friedenberg, and Marciano Siniscalchi. 2016. "Lexicographic beliefs and assumption." *Journal of Economic Theory* 163:955–985.

Dekel, Eddie and Marciano Siniscalchi. 2015. *Epistemic game theory*. Elsevier.

Eaton, Jonathan and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* 48 (2):289–309.

Keisler, Jerome and Byung Soo Lee. 2023. "Common Assumption of Rationality." *SSRN Working Paper* .

Lee, Byung Soo. 2016. "Admissibility and assumption." *Journal of Economic Theory* 163:42–72.

Lorenzoni, Guido and Iván Werning. 2019. "Slow Moving Debt Crises." *American Economic Review* 109 (9):3229–3263.

Mäder, Nicolas. 2024. "Financial Crises as a Phenomenon of Multiple Equilibria and How to Select Among Them." *Journal of Money, Credit, and Banking* 56 (2-3):517–536.

Pearce, David G. 1984. "Rationalizable Strategic Behavio and the Problem of Perfection." *Econometrica* 52 (4):1029–1050.

Stiglitz, Joseph E. and Andrew Weiss. 1981. "Credit Rationing in Markets with Imperfect Information." *American Economic Review* 71 (3):393–410.

Watson, Joel. 1993. "A "Reputation" Refinement without Equilibrium." *Econometrica* 61 (1):199–205.

Yang, Chih-Chun. 2015. "Weak assumption and iterative admissibility." *Journal of Economic Theory* 158:87–101.