Would an Earlier Inception of OMT by the ECB Have Prevented the 2012 Greek Default?

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Abstract

To avert further debt crises following the Greek default of 2012, the European Central Bank (ECB) adopted outright purchases of sovereign bonds as part of its monetary policy regime. This paper examines whether an earlier inception of such purchases (OMT) could have prevented the observed Greek repudiation. To account for the extraordinary circumstances surrounding the Greek default, I construct a novel model of sovereign finance in which default is political and investors' reliance on external credit ratings gives rise to slow moving crises. Estimating the model with Greek data, I find that an earlier inception of OMT plausibly could have prevented the observed default, but the resulting counterfactual Greek state would have been so fragile that, absent any further fiscal consolidation, eventual default was effectively inevitable. Moreover, the present Greek state remains sufficiently fragile that a quick return to a predominantly private financing scheme is not advisable.

Keywords: Greek debt crisis, political default, purification, bounded rationality, nonlinear dynamics, particle filter

JEL codes: H63, G12, E52

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1 Introduction

Following the Great Recession, growing fears of an impending Greek default lead to an extraordinary divergence between Greek and German yields from initial parity to over eighty percent (Figure 1). In turn, following the actual realization of default in March of 2012, the European Central Bank (ECB) announced that it would, if necessary, start purchasing European sovereign debt via so-called Outright Monetary Transactions (OMT), an extraordinary move motivated by the notion that sovereign debt crises can be self-fulfilling,

"We are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium', namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening [...] to 'break' these expectations."

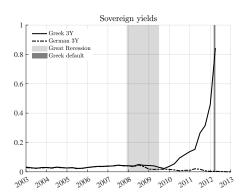
Mario Draghi, July 2012

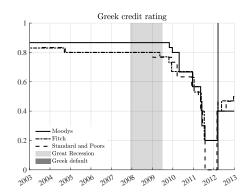
Following the logic outlined above, this paper examines whether a counterfactual earlier inception of OMT — such as after the first credit risk downgrade by Standard & Poors in early 2009 (see Figure 1) — might have sufficed to usher Greece onto a 'good equilibrium' path and, thus, effectively prevented the country's observed default. To this end, I construct a novel model of sovereign finance emphasizing two specific aspects of the Greek crisis: the slow moving run-up and the political nature of default.

A key feature of the Greek crisis was that, prior to default, credit risk — or at least markets' perception thereof — deteriorated steadily, but gradually between December 2009 and March 2012. Thus, to account for this slow moving run-up within the proposed framework, I start by introducing expectational inertia into an otherwise standard model of asset pricing. For this, I specifically assume that investors, rather than assessing credit risk themselves, rely on external credit ratings issued by a rating agency. Aside from accounting for the Greek crisis' slow moving run-up, this assumption provides a natural explanation for the fact that credit ratings have been found to affect secondary market yields and, perhaps most importantly, it theoretically rationalizes the inception of OMT by the ECB.¹

¹Even if activated, OMT is ineffective under rational expectations because it only affects secondary market pricing, but not primary market outcomes.

Figure 1. The Greek crisis





Notes: Figure 1 depicts the recent Greek debt crisis, as captured by secondary market yields (Panel A) and sovereign credit ratings (Panel B), leading up to default in March 2012. Evidently, the crisis evolved gradually over the course of multiple years with each major credit rating agency downgrading Greek debt at least seven times. The depicted credit ratings were constructed by translating the alphabetical ratings into thirty equidistant steps from zero to one.

Aside from its slow moving run-up, a second key feature of the Greek crisis was that all default negotiations were carried out among Greece's creditors themselves (Zettelmeyer et al., 2013). Indeed, with private financing "practically impossible" as early as April of 2010 (Bank of Greece, 2014), the decision if and when Greece would default had shifted from the Greek government to the European Troika (i.e. European Commission, ECB, IMF) well before the observed default. Thus, any theoretical framework whose principal aim is to rationalize the extraordinary dynamics shown in Figure 1 must, first and foremost, provide insights into the objectives and constraints of the European Troika.

To formally incorporate the fact that the observed Greek default was effectively agreed upon at an EU summit in late 2011, the proposed framework explicitly models default as a political decision. More precisely, default is modeled as resulting from a non-cooperative game between an incumbent policy maker and their political base. In this context, nominally noncontigent sovereign debt is viewed as politically contingent in the sense that default may be perceived as excusable if it is "justifiably associated with implicitly understood contingencies" (see Grossman and van Huyck, 1988).² In particular, I assume that the political excusability of default is increasing in the fraction of a country's tax income that is spent on unproductive interest outlays, thus strengthening the policy maker's incentives to default when such outlays are high.

²Grossman and van Huyck (1988) propose that nominal noncontigency may be less relevant for sovereigns than for firms because sovereign debt, unlike corporate debt, is not subject to external enforcement. Thus, sovereign defaults are unlike corporate defaults in that corporate defaults are effectively forced, whereas sovereign defaults, or at least their timing, typically occur strategically (e.g. when they are excusable).

Estimating the proposed theory with Greek data, I find that it performs well in accounting for both the 'stationary' yield phase until 2009 as well as the 'explosive' phase thereafter (Figure 7A). In turn, the estimated model suggests that a counterfactual inception of OMT in 2009 indeed could have prevented the observed default, but even in the event that credit risk had remained low, the counterfactual Greek state would have been so fragile — due to multiple steady states — that eventual default was effectively inevitable absent any further fiscal consolidation (Figure 10).³ Nevertheless, to the extent that additional time could have been leveraged for purposes of such consolidation, the presented analysis supports the proposition that OMT can serve as an effective measure to combat a looming sovereign debt crisis.

In a second series of empirical tests, the estimated model is leveraged to estimate when the observed Greek financing scheme initially became financially unsustainable. I find that the Greek state became financially unsustainable at least six months prior to exhibiting 'mildly explosive' behavior (Figure 8). During the latency period — when fundamentals were financially unsustainable but yields still appeared stationary — the Greek Treasury benefitted from a decrease in the risk free rate and a sequence of positive market perception shocks. However, both of these mitigating factors were insufficient to offset the ever-rising debt level, which continued to advance the country into deeper territories of the financially unsustainable region of its state space. In a final counterfactual, I find that current Greek fundamentals are financially sustainable, but fragility remains extremely high such that promoting a quick return to a predominantly private financing scheme is hardly advisable (see Section 5).

Related literature. This paper contributes to the literature on sovereign default by quantitatively rationalizing the extraordinary evolution of Greek yields depicted in Figure 1. To do so, the presented model explores why sovereign yields may only rise gradually prior to default and, more generally, why they may not instantly reflect innovations in a country's fundamentals. This addresses an important issue raised by Bocola and Dovis (2019), who lament that their "model has hard time capturing the jump in [Italian] spreads observed in the third quarter of 2011 with the fundamental shocks because [fundamentals] barely moved".⁴ This merits further investigation

³This is because the counterfactual Greek path would have remained near an unstable steady state separating the favorable mean-reverting regime from the dreaded explosive regime. Even small perturbations would thus have sufficed to trigger an explosion in yields (at a later time).

⁴The observed rise in yields is attributed to measurement error of the yields themselves.

because any extraordinary rise in spreads is precisely what a sovereign default model ought to be able to explain, either endogenously or with reference to exogenous shocks.

In the proposed theory, the reason why sovereign yields may rise even in absence of fundamental innovations is that investors' credit risk assessments are subject to frictions. Specifically, rather than assessing credit risk themselves, investors are assumed to rely on external credit ratings, the latter of which are based on the prevailing (instead of future) borrowing costs. In effect, rather than featuring multiple equilibria, the resulting theory features multiple steady states. In this setting, small perturbations may suffice to propel a country across an unstable tipping point, thus triggering a gradual, self-reinforcing feedback loop between rising borrowing costs and rising likelihoods of default.⁵ The resulting *intertemporal* debt crises represent a departure from the literature, where default is typically viewed as self-fulfilling in that it is caused by *intratemporal* coordination failures — typically due to a sunspot — among primary market bidders.^{6,7}

The key modeling ingredient that turns self-fulfilling crises into self-reinforcing ones is bounded rationality. Specifically, I assume that investors are rational in that they successfully derive the optimality condition corresponding to their optimization problem, but their rationality is bounded in that informational barriers preclude them from accurately evaluating the derived condition. Of course, the underlying premise that humans are not as informationally and computationally adept as their canonical theoretical counterparts is not new.⁸ Indeed, the empirical sovereign debt literature has found that changes in credit ratings, unanticipated and anticipated, often materially influence observed secondary market yields.⁹ Incidentally, the prevalence of bounded rationality

⁵Conversely, a small initial perturbation in the opposite direction may be sufficient to avert a looming yield spiral. ⁶Extending Calvo's two-period model to the infinite horizon, Alesina et al. (1990) and Cole and Kehoe (2000) show a sovereign's inability to commit to repayment can yield multiple, self-fulfilling equilibria. More recent examples of indeterminate models of sovereign finance include include Conesa and Kehoe (2017), Lorenzoni and Werning (2019), and Bocola and Dovis (2019).

⁷Sovereign default appears to be one of only few strands of macroeconomics in which the existence of multiple equilibria is widely accepted. In fact, after proving that equilibrium in the canonical Eaton-Gersovitz (1981) model is unique, Auclert and Rognlie (2016) go so far as to interpret their result as a shortcoming of the model rather than as a vindication of the uniqueness proposition: "Our objective is not to deny that sovereign debt markets can be prone to self-fulfilling crises, or that OMT may have ruled out a bad equilibrium. Instead, we hope that our results may help sharpen the literature's understanding of the assumptions that are needed for such multiple equilibria to exist."

⁸In a seminal contribution, Simon (?) asserts that "the task is to replace the global rationality of economic man with the kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man." Recent surveys of bounded rationality include Harstad and Selten (2013) and Rabin (2013). As in Rabin (2013) but unlike in Harstad and Selten (2013), bounded rationality and optimization are not viewed as mutually exclusive here.

⁹Under rational expectations, both unanticipated rating changes and the anticipation of rating changes are perfectly inconsequential because only new fundamental information regarding the likelihood of default or loss given default affects true credit risk. In contrast, early studies by Cantor and Packer (1996) and Reisen and von Maltzan

also yields a rationale for why purchases on the *open market* (i.e. OMT) present a viable monetary strategy in the first place.¹⁰

Finally, I also depart from the literature when considering the policy maker's decision to default. For this, existing work typically appeals to a tradeoff between the benefits of lowering the interest burden of debt versus the costs of a temporary fall in output.¹¹ This modeling choice is seemingly supported by the observation that when default is imminent, debt-to-GDP is high, sovereign yields reach their peak, and output contracts (Mendoza and Yue, 2012). However, using quarterly instead of annual data, Levy-Yeyati and Panizza (2011) document that while sovereign defaults typically do coincide with output declines, the latter actually precede the former and so defaults effectively "mark the beginning of economic recovery".¹² Rather than economic, I thus follow Grossman and van Huyck (1988) in assuming that the primary costs of default are political.¹³ Although this modeling choice is particularly apt in the case of Greece, where the decision to default had effectively shifted from the Greek Treasury to the European Troika over the course of 2010, the findings of Levy-Yeyati and Panizza (2011) suggest that this approach may in fact be appropriate more generally.

⁽¹⁹⁹⁹⁾ found that roughly two thirds of all studied rating announcements materially affected sovereign spreads. More recently, Afonso et al. (2012) and Binici et al. (2018) find that announcements still affect spreads. Both show that negative announcements induce stronger effects, but conflicting evidence is presented as to whether effects have become stronger or weaker over time. Arguably, the mere existence of credit ratings is sufficient to motivate the assumption of bounded rationality.

¹⁰Under rational expectations, open market purchases are irrelevant because secondary market yields do not materially influence a country's economic fundamentals.

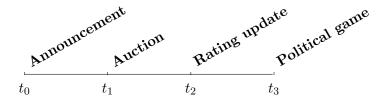
¹¹See Eaton and Gersovitz (1981), Cole and Kehoe (1996), Aguiar and Gopinath (2006), Yue (2010), Conesa and Kehoe (2017), and Na et al. (2018). Cole and Kehoe (2000) and Mendoza and Yue (2012) motivate default-induced output losses via contractionary TFP, Sosa-Padilla (2018) via contractionary bank lending. Bocola and Dovis (2019) also assume output losses, albeit only implicitly via lost tax revenues.

¹²In fact, "growth rates in the post-default period are never significantly lower than in normal times" (Levy-Yeyati and Panizza, 2011).

¹³It is tempting to interpret the observation that recessions precede defaults as evidence that recessions cause default, and not vice versa (post hoc ergo propter hoc). However, Hérbert and Schreger (2017) show that an exogenous increase in sovereign default probabilities does in fact cause lower equity returns, which suggests that the anticipation of default can cause recessions (such anticipation effects are the main focus of study in Bocola (2016)). But, as pointed out by Levy-Yeyati and Panizza (2011), if the referenced anticipation effects were the sole cause of the output losses typically observed during times of default, then the corresponding contemporaneous decision to default becomes trivial. This is because most, if not all, of the output costs associated with default have already been borne through anticipation effects such that the ultimately relevant marginal costs are either very low or zero. In either case, motivating a government's decision to repay via output losses is thus misleading.

2 A model with slow moving crises and political defaults

Each period, the proposed model evolves according to the following timeline. First, following the exogenous realization of external financing needs and the risk free benchmark rate, the treasury announces the issuance of a new bond to cover the government's (expiring) liabilities. Second, primary market dealers are invited to submit bids to purchase the new bond in a silent auction. In valuing the bond, primary market dealers are assumed to lack the necessary information to derive a probabilistically accurate credit risk assessment and, thus, resort to relying on an external assessment issued by a rating agency. Third, following the auction, the rating agency updates its credit rating and secondary market trading ensues. Finally, policy makers enter into a non-cooperative game with the political base to determine whether default occurs or not.



 t_0 : The bond announcement

At the beginning of each period, the treasury observes the exogenous realization of the government's primary deficit x, real economic growth g, and the risk free interest rate r. In turn, present external financing needs (EFN) are calculated as,

$$d \equiv \frac{V}{1+g} + x$$

where V is the face value of an expiring bond that was issued in the previous period (in prior GDP terms). Thus, to avoid immediate default, the proceeds l from the upcoming auction of a new bond with face value V' must satisfy $l \geq d$. Assuming that avoidable defaults with l < d carry severe political repercussions and that the treasury itself does not benefit from issuing any additional bonds, the treasury's objective is given by,

$$\min_{V'} V' \quad \text{s.t.} \quad l \ge d$$

where the proceeds l are determined by the not-yet-known primary market yield y^P . However, since the treasury understands that investors gauge credit risk by way of the prevailing, external

credit risk assessment $\lambda^P \in [0, 1]$, it correctly anticipates $y^P = g_y(r, \lambda^P) \ge r$ via a known mapping g_y to be derived shortly. Thus, the solution to the treasury's problem is given by,

$$l = d \implies V' = d[1 + g_y(r, \lambda^P)]$$

where d, r, and λ^P are all predetermined.

 t_1 : The auction

To derive g_y , let us consider the bond's auction. In the auction, a set of primary market dealers submit bids to invest in the new sovereign bond with face value V' and uncertain payoff $\hat{V'} \leq V'$. All wealth not allocated to the risky bond is diverted towards a risk free asset with a guaranteed return r. Investors are assumed to have a risk neutral objective and thus aim to maximize,

$$\max_{p,q} q^* \mathbb{E}[\hat{V}'] + (m - p^* q^*)(1+r)$$

where m is pre-existing wealth and (q^*, p^*) are the ultimately allocated quantity and price. The above objective implies that the security is fundamentally valued at $\mathbb{V} \equiv \frac{\mathbb{E}[\hat{V}']}{1+r}$. Thus, so long as no winning bid can influence the ultimately transacted rate, bidding $p = \mathbb{V}$ represents a weakly dominant strategy for each investor.¹⁴ Although deriving this policy is simple, *implementing* it may be non-trivial. To see this, let us consider the factors that determine the bond's credit risk:

$$\mathbb{V} = \mathbb{E}[\hat{V}']/(1+r)$$

$$= [(1-\pi)V' + \pi\gamma V']/[1+r]$$

$$= \left(\frac{1-\lambda}{1+r}\right)V'$$

where credit risk λ consists of the probability of default π and the loss given default $1-\gamma$, both of which realistically depend on the government's prevailing borrowing costs and, thus, on the outcome of the auction itself. Indeed, determining (auction-implied) credit risk is sufficiently difficult that real-world financial markets feature institutions — rating agencies — whose sole purpose is to gauge λ . Following this logic, I assume that investors are boundedly rational in that they are unable to accurately map the effects of their (individual and collective) behavior into the policy maker's decision to default. In particular, for lack of better information, investors replace

¹⁴To ensure that such bidding is in fact optimal, I assume that investors collectively have 'deep pockets' and that the new bond is sold in the format of a multi-unit analogue of a single-unit second-price auction. That is, the ultimately transacted price is set equal to the highest bid among all non-winning bidders.

the 'fully rational' object \mathbb{V} with the readily available, behavioral object \mathcal{V} ,

$$\mathcal{V} \equiv \left(\frac{1 - \lambda^P}{1 + r}\right) V'$$

where λ^P is the *most recent* credit rating issued by the agency. In effect, since the bond's value is perceived to be \mathcal{V} , bidding $p = \mathcal{V}$ is perceived as a weakly dominant strategy by each investor. In aggregate, this implies $p^* = \mathcal{V}$ and, thus,

$$y^{P} = \underbrace{\left(\frac{1+r}{1-\lambda^{P}}\right) - 1}_{g_{y}(r,\lambda^{P})}$$

 t_2 : The rating update

Following the auction, the credit rating agency observes (V', y^P) and — leveraging its knowledge of the policy makers' objectives and constraints — updates its credit risk assessment to the auction-implied, true credit risk $\lambda^{P'} = \lambda = g_{\lambda}(V', y^P)$. In turn, following the rating update, the Walrasian auctioneer proceeds by equilibrating secondary market demand with the bond's fixed supply via the secondary market price $p^S = \frac{V'}{1+y^S}$. In equilibrium, we have,

$$y^{S} = \underbrace{\left(\frac{1+r}{1-\lambda^{P'}}\right) - 1}_{q_{y}(r,\lambda^{P'})}$$

such that observed secondary market yields reflect the true credit risk λ . Finally, to illuminate the source of said risk, I now turn to discussing the non-cooperative game between the incumbent policy maker and the country's political base.

t_4 : A political game

In the proposed model, default results from a political game between the incumbent policy maker and the country's political base at the end of each period. The game's strategy space is given by $\Sigma^G \times \Sigma^B = \{\text{default}, \text{not default}\} \times \{\text{reelect}, \text{not reelect}\}$. Across this space, preference orderings are fixed except when the policy maker is reelected. In this case, when the fraction of output spent on unproductive interest outlays (i.e. $e \equiv dy^P$) is below a certain threshold \underline{e} , the policy maker prefers not to default. Conversely, when said fraction is above \underline{e} , the policy maker prefers to default. Finally, if default occurs, its excusability as perceived by the political base is

assumed to be increasing in the interest rate burden of debt, i.e. $b_3 - b_4 = g_e(e)$ with $g_e > 0, g'_e < 0.15$

Figure 2. The political game

Policy maker

Base

(b_3, g_4)	(b_2, g_3)	Not reelect
(b_4,g_k)	(b_1,g_l)	Reelect

Default Not Default

Notes: Figure 2 depicts the strategic interaction between an incumbent policy maker and their political base with $g_i > g_j, b_i > b_j$ when i < j. Moreover, when $e \le \underline{e}$, we have (k = 2, l = 1), and when $e > \underline{e}$, (k = 1, l = 2). That is, so long as interest outlays are sufficiently low, the policy maker prefers, at least weakly, not to default. Conversely, if interest outlays are high, not defaulting is only preferred if the reelection fails (so as to 'pass the buck').

To derive empirical predictions from the depicted game, we must distinguish between the two cases. First, when $e \leq \underline{e}$, not defaulting is (at least) weakly dominant for the policy maker such that relatively weak epistemic assumptions — mutual knowledge of rationality (and, if $e = \underline{e}$, caution) — are sufficient to uniquely induce the 'good' no-default/reelect outcome. In turn, when $e > \underline{e}$, the referenced equilibrium in pure strategies disappears, but a unique, mixed-strategy equilibrium emerges in its place,

$$\Pr(\sigma^G = \text{default}) = \frac{b_1 - b_2}{(b_1 - b_2) + (b_3 - b_4)}$$
$$\Pr(\sigma^B = \text{not reelect}) = \frac{g_1 - g_2}{(g_1 - g_2) + (g_3 - g_4)}$$

Thus, when $e > \underline{e}$, our primary object of interest — the likelihood of default — depends on the utility of the political base across the various outcomes of the game. In particular, it increases in the difference $b_1 - b_2$, but decreases in $b_3 - b_4$. To understand these properties, note that said differences represent the political base's strength of preference for rewarding (in case of no-default) and punishing (in case of default) the policy maker. Thus, the reason that the likelihood of default rises during times when default is perceived as excusable lies in the political base's muted desire to punish. In effect, the key assumption that establishes the desired causal link between higher borrowing costs and higher probabilities of default is $g'_e < 0$.

Given the main result of the political game — that the policy maker optimally mixes between default and not default — a conceptual point is in order. In the described context, mixed strategies are not to be interpreted as acts of literal randomization, but rather as pure strategies that appear

¹⁵For a more detailed discussion of the chosen preference rankings, see Appendix C.

random from the point of view of an outside observer. For example, when the policy maker defaults, this choice need not be random. Instead, it may represent an optimal response to a small shock nudging the policy maker towards default.¹⁶

 t_4 : Default

If the political game leads to default, the treasury's obligations are reduced to a stochastic fraction of GDP ξ with the implied haircut being enforced uniformly across all investors.

$$\hat{V}' = \begin{cases} V' & \text{if no default} \\ \xi & \text{in default} \end{cases}$$

We thus have $\gamma = \min\{1, \xi/V'\}$ such that the credit risk function used by the rating agency can be derived as follows,

$$\lambda = \pi(1 - \gamma)$$

$$= \underbrace{\frac{\left(1 - \min\{1, \xi/V'\}\right)}{1 + g_e\left(\frac{y^P V'}{1 + y^P}\right)}}_{q_\lambda(y^P, V', \xi)}$$

where, without loss of generality, $b_1 - b_2$ was set to unity.¹⁷ Finally, the model is closed by imposing a distribution for ξ and a functional form for the political excusability function g_e . For this, I assume that g_e is isoelastic, i.e. $g_e(e) = \beta(e - \underline{e})^{-\alpha}$ with $\alpha > 0, \beta > 0$ when $e > \underline{e}$, and that post-default debt ξ is distributed logit-normally in $(0, 2\bar{\xi})$. That is, $\xi = \frac{2\bar{\xi}}{1 + \exp(-\varepsilon^{\xi})}$ where ε^{ξ} is Gaussian.

3 Dynamical system representation

Unlike most macroeconomic models, the proposed theory features a closed-form representation of its implied dynamical system. In particular, the economy is governed by the following set of equations,

¹⁶Indeed, following the steps of Harsanyi's famous 'purification theorem' (see Harsanyi, 1973), it is possible to construct an incomplete information analogue of the described, complete information game, whereby the policy maker defaults if and only if default yields (random) utility in excess of some critical threshold. In this case, as the variance of such noise vanishes, there exists a sequence of pure strategy Nash equilibria that converges arbitrarily closely to the above referenced equilibrium in mixed strategies.

¹⁷This is without loss of generality so long as g_e is scaled proportionately.

$$y_t^S = \frac{1 + r_t}{1 - \pi_t (1 - \gamma_t)} - 1 \tag{1}$$

$$\pi_t = \frac{1}{1 + \max\{0, \beta(d_t y_t^P - \underline{e})^{-\alpha}\}}$$

$$\tag{2}$$

$$\gamma_t = \min\left\{1, \frac{\xi_t}{d_t(1+y_t^P)}\right\} \tag{3}$$

$$y_t^P = \left(\frac{1+r_t}{1+r_{t-1}}\right)(1+y_{t-1}^S) - 1 \tag{4}$$

where the fact that last period's secondary market yield and the risk free rate constitute state variables is indicative of investors' boundedly rationality. Further notice that our system is entirely parameterized by $\theta = (\underline{e}, \alpha, \beta, \overline{\xi})$ and that — since the endogenous state is one-dimensional — dimensionality can principally be reduced to one by plugging (2)-(4) into (1),

$$y_{t}^{S} = \frac{1 + r_{t}}{1 - \min\left\{1, \frac{\xi_{t}}{d_{t}\left(\frac{1 + r_{t}}{1 + r_{t-1}}\right)\left(1 + y_{t-1}^{S}\right)}\right\}} - 1$$

$$1 - \frac{1 - \min\left\{0, \beta\left[d_{t}\left[\left(\frac{1 + r_{t}}{1 + r_{t-1}}\right)\left(1 + y_{t-1}^{S}\right) - 1\right] - \underline{e}\right]^{-\alpha}\right\}}{1 + \max\left\{0, \beta\left[d_{t}\left[\left(\frac{1 + r_{t}}{1 + r_{t-1}}\right)\left(1 + y_{t-1}^{S}\right) - 1\right] - \underline{e}\right]^{-\alpha}\right\}}$$
(1')

Equation (1') is 'simple' in the sense that it is one-dimensional and that it is available in closedform, but it is visibly nonlinear and so even small changes in initial conditions can potentially cause large swings in asymptotic behavior. To illustrate more concretely the implications of these nonlinearities, I now proceed by defining *financial sustainability*.

Financial sustainability

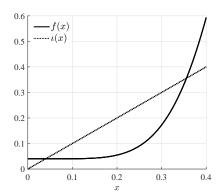
From the point of view of policy, a counterfactual of particular interest is whether a particular fundamental state is *financially sustainable*. That is, how would borrowing costs evolve if, going forward, the debt level and the risk free rate remained at their present levels.¹⁸ To formalize this notion of financial sustainability, I construct a new, auxiliary system by repeatedly iterating on (1') while setting $d_t = d_0$, $r_t = r_0$, and $\xi_t = \bar{\xi}$ for each $t \geq 0$ and some initial condition (y_0^S, d_0, r_0) . This yields the following 'reduced' difference equation,

 $^{^{18}}$ This counterfactual does not describe how yields will actually evolve, which is determined by (1'), but it gives policy makers an idea how they would evolve if the government managed to stabilize them at their current levels.

$$y_{t}^{S} = \underbrace{(1+r_{0}) \left(\frac{1+\max\{0, \beta[d_{0}y_{t-1}^{S} - \underline{e}]^{-\alpha}\}}{\min\{1, \overline{\xi}/d_{0}(1+y_{t-1}^{S})\} + \max\{0, \beta[d_{0}y_{t-1}^{S} - \underline{e}]^{-\alpha}\}} \right) - 1}_{f(y_{t-1}^{S}|d_{0}, r_{0}; \theta)}$$
(5)

with corresponding interest outlays in the amount of $e_t = d_0 y_t^P = d_0 y_{t-1}^S$ for each $t > 0.^{19,20}$ Just like (1'), (5) is simple in that it is one-dimensional and that it is available in closed-form, but it remains highly nonlinear such that small changes in initial conditions can potentially cause large swings in asymptotic behavior. For example, Figure 3 depicts f taking as given the fundamental state $(d_0, r_0) = (1, 0.01)$ and the parameter vector $(\alpha, \beta, \underline{e}, \overline{\xi}) = (2, 0.01, 0.05, 1)$.

Figure 3. Cobweb: A sample function f that governs the difference equation (5)



Notes: Figure 3 depicts the difference equation (5) for the initial condition $(d_0, r_0) = (1, 0.01)$, thus illustrating two points. First, $f(y|d_0, r_0; \theta)$ is highly nonlinear. Second, the nonlinearity of f gives rise to two separate steady states, only one of which is stable. Once yields pass through the unstable threshold near y = 0.36, they begin to diverge.

The primary insight from Figure 3 is that the nonlinearities in f can give rise to multiple steady states. Specifically, for the given parameterization and the chosen macroeconomic state, (5) features a stable steady state in the vicinity of y = 0.03 and an unstable steady state near y = 0.36. Therefore, any initial condition $y_0 \in [0, 0.36)$ asymptotically maps towards the preferred steady state $\lim_{t\to\infty} y_t \approx 0.03$, whereas any initial condition $y_0 \in (0.36, \infty)$ induces asymptotic divergence. In effect, the unstable steady state near y = 0.36 constitutes a tipping point beyond which yields diverge. We are then ready to formally define financial sustainability.

¹⁹This counterfactual abstracts from default itself and thus should be viewed as describing the counterfactual evolution of yields prior to default only.

²⁰Further notice that the numerator in the large fraction of (5) weakly exceeds the denominator and so we must have $y_t^S \ge r_0 \ \forall t \ge 0$.

Financial sustainability. A state $X_0 = (y_0^S, d_0, r_0)$ is said to be \bar{e} -financially sustainable if and only if repetitively iterating on (5) given the initial condition X_0 yields $\lim_{t\to\infty} e_t \leq \bar{e}$, where \bar{e} is a predetermined fraction of output that a government is willing to (permanently) spend on 'unproductive' interest outlays.²¹

The proposed notion of financial sustainability is conditional in two ways. First, it conditions on a permanent fraction of output that a government is willing to spend on interest outlays. Second, and more importantly, it also conditions on the premise that the debt level and interest rates indefinitely remain at their initial levels. The practical implications of this second assumption are most apparent in the case of financially unsustainable state: A state is financially unsustainable if the economic forces endogenous to financial markets will iteratively carry the state towards 'unacceptable' regions of the state space $\mathcal{X}^E(\bar{e}|\theta) \equiv \{X \in \mathbb{R}^3 \mid dy > \bar{e}\}$ unless the government manages to reduce its per-GDP level of debt — either primary surpluses or economic growth — and/or the risk free rate falls. To the extent that policy making tends to be local (should we reduce debt/lower interest rates?) rather than global (what is the optimal level of debt/interest rates?), this second conditional nature of financial sustainability is precisely what renders it so relevant from a practical perspective.

Given our definition of financial sustainability, we can then partition the state space into the subset of financially sustainable and financially unsustainable states/initial conditions.

$$\mathcal{X}^{S}(\bar{e}|\theta) \equiv \left\{ X_0 \in \mathbb{R}^3 \mid \lim_{n \to \infty} d_0 f^n(y_0|d_0, r_0; \theta) \le \bar{e} \right\}$$
 (6)

$$\mathcal{X}^{U}(\bar{e}|\theta) \equiv \left\{ X_0 \in \mathbb{R}^3 \mid \lim_{n \to \infty} d_0 f^n(y_0|d_0, r_0; \theta) > \bar{e} \right\}$$
 (7)

where f^n is the *n*th iterate of f. The sets generated by (6) and (7) each represent to a basin of attraction mapping to their respective steady state (or to no steady state if yields diverge). To recover \mathcal{X}^S and \mathcal{X}^U , we can solve (5) for its fixed points $\mathcal{P}(d_0, r_0)$. By definition, we have,

$$y^* = f(y^*|d_0, r_0; \theta) \tag{8}$$

for any $y^* \in \mathcal{P}(d_0, r_0)$. Partitioning the state space into financially sustainable and unsustainable states is helpful in highlighting the economic relevance of initial conditions. For this, d_0 and

²¹One could principally define fiscal sustainability or monetary sustainability analogously, namely by studying the parallel fiscal and monetary subsystems. For example, abstracting from default, Bohn (1998) famously established that debt levels are strictly stationary if primary deficits fall more than linearly as debt levels rise.

 r_0 are best thought of as 'parameters' such that we can examine our auxiliary system's asymptotic behavior by way of a bifurcation diagram. For example, consider Figure 4 which depicts two sets of fixed points of f. In Panel A, I vary the debt level d_0 while the risk free rate is held fixed at $r_0 = 0.03$, whereas in Panel B, I vary r_0 while the debt level is held fixed at $d_0 = 1$.

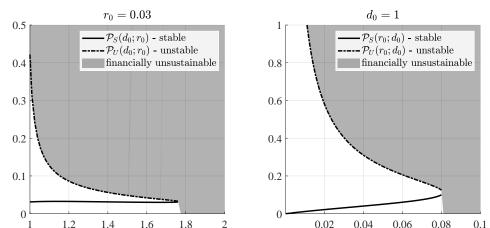


Figure 4. Bifurcation diagram: financially sustainable vs. financially unsustainable states

Notes: Figure 4 depicts the correspondence $\mathcal{P}: \mathbb{R}^2 \rightrightarrows \mathbb{R}$ which maps the macroeconomic fundamentals (d,r) into fixed points of f. All states in the sustainable region \mathcal{S} (white) asymptotically converge towards the preferred stable steady state, whereas all points in the unsustainable region \mathcal{U} (gray) asymptotically diverge.

 d_0

The key insight from Figure 4 is that higher debt levels and risk free interest rates are mirrored by a lower tolerance for yields as measured by the good steady state's distance from its unstable counterpart $\mathcal{P}_U(d_0, r_0) - \mathcal{P}_S(d_0, r_0)$. That is, higher debt levels and a higher risk free rate render a government's financing scheme more fragile, a proposition strongly supported by economic intuition. Therefore, to the extent that we can think of the distance $\mathcal{P}_U(d_0, r_0) - \mathcal{P}_S(d_0, r_0)$ as a stable steady state's resilience to perturbations, Figure 4 illustrates that a government's financing scheme loses resilience as macroeconomic fundamentals deteriorate.²² For example, in the depicted bifurcations near $d_0 \approx 1.75$ and $r_0 \approx 0.08$, the preferred stable node collides with the unstable threshold and, after briefly forming a saddle, ceases to exist altogether. This occurs as f in Figure 3 shifts upward and loses its intersections with the identity function ι .

To gain a more encompassing view of resilience, Figure 5 partitions the space of macroeconomic states into three regions: infinite tolerance, finite tolerance, and zero tolerance. In the infinite tolerance region, the initial condition y_0 is irrelevant because there is a unique, stable steady

²²See Holling (1973) for a discussion of different types of 'resilience' in the context of dynamical systems.

state with low probability of default. In the finite tolerance region, the preferred steady state is complemented by an unstable steady state, beyond which yields diverge. In this region, the initial condition y_0 crucially determines the system's asymptotic behavior. Lastly, in the zero tolerance region, initial conditions are once again inconsequential because yields diverge regardless of y_0 .

Figure 5. Tolerance for yield

Notes: Figure 5 partitions the macroeconomic state space into separate regions of tolerance for yield. In the infinite tolerance region, the initial condition y_0 is irrelevant because we have a unique, stable steady state with insignificant levels of credit risk. In the finite tolerance region, the good steady state is complemented by an unstable steady state, or threshold, beyond which yields asymptotically diverge. In this region, tolerance for yield is finite and infinitesimally small changes in initial conditions can dramatically alter asymptotic behavior. Lastly, in the zero tolerance region, y_0 is once again inconsequential because yields will diverge regardless of initial conditions.

The key insight in Figure 5 is that resilience can be highly sensitive to small changes in macroe-conomic fundamentals, especially along the debt dimension. For example, if the risk free interest rate is high — suppose $r_0 = 0.08$ — a seemingly minuscule change in external financing needs from just below 100% of GDP to just above 100% of GDP is sufficient to carry the state from the infinite tolerance region through the finite tolerance region into the zero tolerance region. Moreover, notice that the finite tolerance subset of the state space may be thought of as a financially fragile region with a potential for self-reinforcing crises: If current yields are low, they will converge to the favorable steady state, whereas if they are high, they will diverge.

Although the nonlinearities in (1') are most evident when they manifest themselves in the form of multiple steady states and bifurcations such as the ones depicted in Figures 2-4, the main takeaway from the discussion thus far lies in the more general observation that even small changes in initial conditions can generate a wide range of asymptotic behavior (see Lorenz, 1963), an insight that will be helpful to explain the extraordinary evolution in Greek yields depicted in Figure 1.

4 The Greek crisis

In this section, I estimate the model with Greek data in order to conduct a series of counterfactuals. To estimate $\theta = (\underline{e}, \alpha, \beta, \overline{\xi})$, I first rewrite (1') in the following state space form,

$$X_t = F(X_{t-1}, \varepsilon_t | \theta), \quad Y_t = X_t + \eta_t$$

where $X_t = (y_{t-1}^S, r_t, d_t)$, $\varepsilon_t = (\varepsilon_t^{\xi}, \varepsilon_t^r, \varepsilon_t^d)$, $Y_t = (\hat{y}_{t-1}^S, \hat{r}_t, \hat{d}_t)$, and $\eta_t = (0, 0, \eta_t^d)$.^{23,24} In particular, it is thus assumed that the only series that is observed with measurement error are the Greek external financing needs, whereas Greek yields and the risk free rate are observed without such error, $\hat{y}_t^S = y_t^S, \hat{r}_t = r_t$. I then proceed by assuming that ε_t^ξ and η_t^d are uncorrelated Gaussian random variables and use their densities to derive a model-implied likelihood with a particle filter as described in Appendix D.²⁵ Finally, to conduct Bayesian estimation, I choose a prior for each parameter. The chosen priors and the corresponding resulting posterior mode (MAP) and the posterior mean (BPM) are reported in Table 1.

TABLE 1. BAYESIAN ESTIMATION

θ	Interpretation	$\hat{ heta}_{MAP}$	$\hat{ heta}_{BPM}$	Prior
α β $e \over \bar{\xi}$	Excusability elasticity Excusability level Excusability threshold Haircut central location	0.448 0.870 0.015 0.829	0.441 0.879 0.015 0.830	$\mathcal{U}(0,5)$ $\mathcal{U}(0,5)$ $\mathcal{LN}(\log(1/60), 0.15)$ $\mathcal{LN}(\log(0.8), 0.1)$

Notes: Table 1 reports the Bayesian posterior mode $\hat{\theta}_{MAP}$ and the Bayesian posterior mean $\hat{\theta}_{BPM}$ recovered from the model-implied likelihood in conjunction with the listed priors. The likelihood is constructed using the particle filter described in Appendix D, which also contains a discussion of the above priors as well as graphs that plot prior against posterior probability. In line with contemporary practice, $\sigma = (\sigma_{\xi}, \sigma_{\eta})$ is set externally. Specifically, I choose $\sigma = (0.08, 0.03)$ such that a $3\sigma_{\xi}$ -shock induces $\xi_t = \bar{\xi} \pm 0.1$ and a $3\sigma_{\eta}$ -error induces $\tilde{d}_t = \hat{d}_t \pm 0.1$ respectively.

Given the similarity between $\hat{\theta}_{MAP}$ and $\hat{\theta}_{BPM}$, all further analysis is conducted with $\hat{\theta} \equiv \hat{\theta}_{MAP}$, the latter of which yields two immediate insights. First, $\hat{\underline{e}} = 0.015$ indicates that spending less than 1.5% of GDP on interest outlays is mirrored by zero credit risk because — in the language of Grossman and van Huyck (1988) — a corresponding credit event would be viewed as "unjustifiable repudiation". Second, since credit risk is also zero whenever $\xi_t \geq V_{t+1}$, the central location $\hat{\xi} \approx$

 $^{^{23}\}mathrm{See}$ appendix E for a discussion of the data used in $Y_t.$

²⁴Technically, X_t also includes the lagged value of the risk free interest rate.

²⁵For purposes of estimating the parameters of interest, specifying a distribution for the shocks $(\varepsilon_t^r, \varepsilon_t^d)$ is not necessary since time series data is available for both d_t and r_t . However, such a specification will be required for purposes of conducting the ensuing counterfactual simulations.

0.83 indicates that debt-to-GDP ratios of 76% and 73% imply zero credit risk at the 97.5% and 99.5% level respectively. The main point here is then that even without knowing the government's objective, investors can — since $e_t \leq \underline{e}$ and $\xi_t \geq V_{t+1}$ each imply zero contemporary credit risk — generate a rough credit risk assessment by way of examining current interest expenses e_t as well as the prevailing per-GDP face value of debt V_{t+1} . Of course, by appealing to our theory, we can produce a more sophisticated, quantitative assessment. Specifically, augmenting $\hat{\underline{e}}$ with $\hat{\alpha}$ and $\hat{\beta}$ yields the model-implied probability of default depicted in Figure 6.

Figure 6. Probability of default as a function of per-GDP interest outlays

Notes: Figure 6 illustrates the estimated model's implied, Greek probability of default $\pi = \hat{g}_{\pi}(e) = [1 + \hat{\beta}(e - \underline{\hat{e}})^{-\hat{\alpha}}]^{-1}$ as a function of interest outlays e. For example, if Greece spent 20% of its GDP on interest, the resulting likelihood of default in said period would be roughly 35%. Default is avoided almost surely as long as $e < \underline{\hat{e}}$.

Figure 6 depicts the model-implied, Greek probability of default as a function of per-GDP interest expenses as determined in the auction. For example, if Greece spent all of its tax income on interest outlays in a given period, $e \approx 0.33$, the corresponding probability of default in said period would be roughly 40%.²⁶

Given the parameterized version of the model, I start by assessing model performance using a particle smoother and then study how the Greek economy would have fared under a variety of counterfactual scenarios.

Model performance

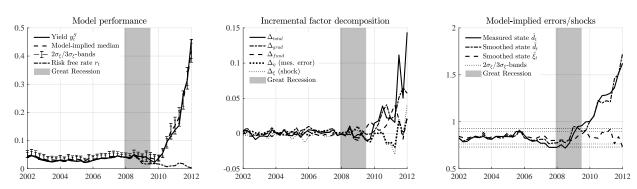
To assess model performance given $\hat{\theta}_{MAP}$, a particle smoother is used to recover an estimate of each period's Greek state as shown in Figure 7.²⁷

 $^{^{26}}$ If such a situation were to persist, say for *n* periods, the probability that we would observe default *in any period* would be much higher and equal $1 - 0.6^n$.

²⁷The 'filtered' state as implied by the particle filter only accounts for past observations, whereas a 'smoothed' estimate exploits all available data. See Godsill, Doucet, and West (2004) for a discussion of the corresponding "forward-filtering, backward smoothing" logic. Using their notation, the particle filter yields $p(X_t|Y_{1:t})$, whereas I

Panel A of Figure 7 plots the observed Greek yield y_t^S against its model-implied conditional distribution as determined by equation (1').²⁸ Overall, the model performs well in that it simultaneously accounts for both the locally stationary phase as well as for the 'mildly explosive' phase starting in early 2010 without appealing to extraordinary shocks and/or measurement errors (Panel C).²⁹ The primary reason that the model is able to endogenously motivate the extraordinary rise in yields is twofold. First, the nonlinearities in (1') are naturally well-equipped to turn small changes in macroeconomic fundamentals — the debt level and the risk free rate — into large swings in yields. Second, inertia in expectation formation allows for a lagged transmission of fundamental innovations to yields (Panel B). In particular, while rational expectations might dictate that yields switch dramatically from one equilibrium to another, inertia in expectation formation translates indeterminacy into multistability, thus turning discrete jumps between multiple equilibria into gradual transitions between multiple steady states.

Figure 7. Model performance, incremental factor decomposition, and model-implied errors/shocks



Notes: The main insight from Figure 7 is twofold. First, Panel A illustrates the model's strong empirical performance by contrasting the observed yield y_t^S against its conditional distribution implied by equation (1'). In turn, Panels B and C provide insights into the various factors that drove the rise in Greek yields between 2010 and 2012. Specifically, Panel B decomposes the incremental change in yields into four factors: gradual/lagging evolution due to bounded rationality Δ_{grad} , change in macroeconomic fundamentals (debt level and risk free rate) Δ_{fund} , measurement error of the debt level Δ_{η} , and shocks to the prospective post-default debt level Δ_{ξ} . Finally, Panel C illustrates the two smoothed states $(\tilde{d}_t, \tilde{\xi}_t)$ recovered via the particle smoother. Evidently, the model neither requires extraordinary measurement errors, nor extraordinary shocks to explain the extraordinary evolution of yields seen in Panel A.

Panel B illustrates the incremental contribution of the following four factors to the observed rise in yields: gradual/lagging evolution due to bounded rationality Δ_{grad} , change in macroeconomic

am interested in $p(X_t|Y_{1:T})$.

²⁸The graph highlights the model-implied median (rather than the mean) because the model-implied distribution is often partially discrete with a large point mass at $y_t^S = r_t$ since any $\xi_t \leq V_{t+1}$ implies a credit risk of zero.

²⁹Recall that Bocola and Dovis (2019) attribute a majority of the locally explosive rise in Italian credit spreads in late 2011 to measurement error.

fundamentals (debt level and risk free rate) Δ_{fund} , measurement error of the debt level Δ_{η} , and shocks to the prospective post-default debt level Δ_{ξ} . The main insight here is that yields gradually adapting to prior fundamental innovations accounts for the majority of the observed rise in yields, whereas contemporary innovations and measurement error only play secondary and tertiary roles. For example, in late 2011, when the effects of the rising Greek debt level was effectively offset by a simultaneous decrease in the risk free rate, yields continued to surge as investors continued to gradually update their expectations.

Finally, Panel C displays the two states $(\tilde{d}_t, \tilde{\xi}_t)$ recovered via the particle smoother. Importantly, the model neither requires much measurement error between the measured debt level \hat{d}_t and the smoothed debt level \tilde{d}_t nor any extraordinary shocks $\tilde{\xi}_t$ to rationalize the observed evolution in yields in Panel A. In particular, notice that the recovered shock series $\tilde{\xi}_t$ appears to follow a similar path — mostly within its $2\sigma_{\xi}$ bands — before and after the Great Recession.

Having assessed model performance, I now turn to constructing a variety of counterfactual Greek states $\{\check{X}_t\}$. Although all variables are computed for each counterfactual, the focus will lie on the counterfactual debt level \check{d}_t and counterfactual secondary market yields \check{y}_t^i . For ease of exposition, I drop the secondary market yield's S superscript and replace it with $i \in \{a, b, c\}$ for each of the three counterfactuals.

When did the Greek state become financially unsustainable?

In a first counterfactual, I leverage the estimated economy to study when the Greek state first became financially unsustainable. That is, I ask how Greek yields would have evolved asymptotically if the government, for each point in time, had managed to stabilize its macroeconomic fundamentals — EFNs and the risk free interest rate — at their respective contemporaneous levels. For this, I leverage the estimated version of (5) to recover a sequence of dynamical systems, one for each pair (\tilde{d}_t, r_t) . In turn, each system generates its own counterfactual evolution of future Greek yields $\check{y}_{\tau \geq t}^a$ as well as a corresponding set of asymptotic fixed points $\check{y}_t^{\star, a_i} \in \mathcal{P}(\tilde{d}_t, r_t)$, $i \in \{1, 2, 3\}$, where $\mathcal{P}(\tilde{d}_t, r_t)$ is recovered by resolving (8) given (\tilde{d}_t, r_t) .

Figure 8 illustrates that until early 2009, the recovered sequence of fix points consists of a seemingly stationary sequence of unique, stable steady states \check{y}_t^{\star,a_1} . However, as Greek fundamentals started to deteriorate during the Great Recession, the permanent fixed point \check{y}_t^{\star,a_1} began to rise and

was occasionally complemented by two other steady states — a stable one \check{y}_t^{\star,a_3} and an unstable one \check{y}_t^{\star,a_2} — near the risk free rate.³⁰ Whenever multiple steady states did exist, Greek yields continued to diverge because they consistently laid in the basin of attraction of \check{y}_t^{\star,a_3} .

Figure 8 indicates that the Greek financing scheme started violating the financial sustainability condition $\tilde{d}_t \tilde{y}_t^{\star,a_1} \leq \bar{e}$ in September of 2009, or two quarters prior to when Phillips and Yu's (2011) date stamping algorithm rejects a unit root vs. 'mildly explosive' behavior.³¹ Thus, the Greek state became financially unsustainable at least half a year prior to exhibiting mildly explosive dynamics.

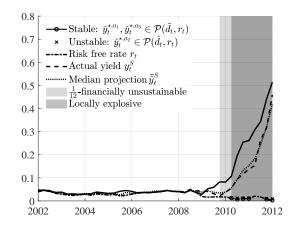


Figure 8. Counterfactual A: Financial sustainability

Notes: Figure 8 plots evolution of Greek yields against the model-implied set of asymptotic fixed points $\mathcal{P}(\tilde{d}_t, r_t)$ derived from equation (5). The main point here is to show why the Greek financing scheme became financially unsustainable precisely when spreads nearly reached zero in late 2009, namely because the joint rise in \check{y}_t^{\star,a_1} and \tilde{d}_t caused a failure of the financial sustainability condition $\tilde{d}_t\check{y}_t^{\star,a_1} \leq \bar{e}$.

To understand why observed Greek yields decreased throughout 2009 while their asymptotic counterpart — the steady state \check{y}_t^{\star,a_1} — increased, notice that the fixed points in $\mathcal{P}(\tilde{d}_t,r_t)$ do not represent an expectation or projection of the present state, which is captured by \tilde{y}_t^S , but rather an asymptotic tendency in a counterfactual world where the prevailing debt level and the risk free remain unchanged for the foreseeable future. Thus, while asymptotic yields rose due to a fundamental deterioration throughout 2009, the reason why observed Greek yields fell was due to a decrease in the risk free interest rate as well as due to favorable credit risk perceptions $\tilde{\xi}_t$ (see

³⁰The reason why two steady states do not exist permanently is that changing fundamentals occasionally cause them to collide in a bifurcation as described in section 2. See Appendix B for an illustration of the evolution of the set $\mathcal{P}(\tilde{d}_t, r_t)$ between 2008 and 2012.

³¹To determine when the Greek state first became financially unsustainable (as seen in Figure 8), I first have to choose a maximum level of GDP that a government is willing to permanently spend on interest outlays \bar{e} . Since \bar{e} is not identified in the data, I conservatively assume $\bar{e} = \frac{1}{12}$ because, realistically, no government is willing to permanently spend a quarter of its tax revenues on interest outlays (Greek tax-to-GDP ratio roughly amounts a third).

Figure 7). In effect, Figure 8 suggests that Greek yields were bound to start increasing as early as December 2008, but the country's Treasury benefitted, albeit temporarily, from a fall in the risk free rate and from a sequence of fortuitous market perception shocks which allowed the impending surge to remain latent.

With the Greek financial state becoming unsustainable as early as late 2009, one might naturally wonder whether an inception of OMT at the time would have sufficed to avert the observed Greek default in 2012.

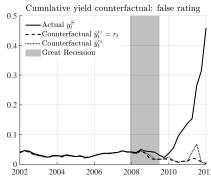
Was the Greek crisis self-reinforcing?

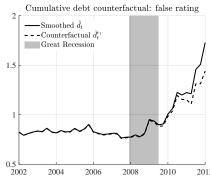
To assess the potential effects of an earlier inception of OMT, I now turn to examining the cumulative effects of a series of counterfactual credit ratings $\check{\lambda}^c_t = 0 \ \forall t.^{32}$ In effect, this approach examines what would have happened to the Greek financial state if markets had artificially continued to view its debt as carrying zero credit risk as they had before. In turn, if the Greek crisis was self-reinforcing, then such (initially) artificial assessments might have, over the course of time, produced a counterfactual Greek state that would have retroactively justified them. And, as can be seen in Figure 9, the estimated model does in fact permit that such a self-reinforcing view of the Greek crisis.

Figure 9. Counterfactual B: Cumulative effects of counterfactual credit ratings

Cumulative vield counterfactual: false rating

Cumulative debt counterfactual: false rating





Notes: Figure 9 depicts the counterfactual evolution of the Greek state assuming that perceived credit risk had remained at zero throughout the entire interval. Trivially, the corresponding yield counterfactual $\check{y}_t^{b_1}$ mirrors the risk free rate while the resulting counterfactual debt series lies below the smoothed series \tilde{d}_t . The main insight then lies in the second counterfactual yield $\check{y}_t^{b_1}$, which accurately accounts for model-implied credit risk while taking as given the counterfactual state. In particular, with the exception of mid 2011, $\check{y}_t^{b_2}$ closely mirrors the risk free rate because low yields would have given rise to a counterfactual Greek state with little to no credit risk. That is, the Greek crisis appears to have been self-reinforcing.

 $^{^{32}}$ Here, the idea is that OMT allows country's to refinance themselves at artificially low rates, which may be modeled in the form of an artificially favorable credit risk perception.

Figure 9 captures the main result of the paper. Specifically, the main result is not that the counterfactual yield $\check{y}_t^{b_1}$ mirrors the risk free rate, which happens by construction, but rather that $\check{y}_t^{b_2}$, which accurately accounts for model-implied credit risk, also closely mirrors the risk free rate. That is, if markets had continued to treat Greek debt as carrying no credit risk during and after the Great Recession, Greek debt would in fact — with the exception of 2011 when a rise in the risk free rate briefly caused the preferred steady state to vanish (Figure 8) — have continued to carry little to no credit risk.

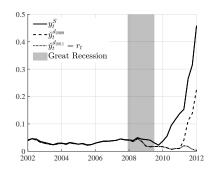
It is tempting to dismiss the counterfactual depicted in Figure 9 as it is based on 'inaccurate' assessments of credit risk. However, notice that the employed particle filter was used to fit the Greek data under the explicit premise, since secondary market trading is fully rational, that the observed yield series accurately reflected the underlying bonds' credit risk. But what if the initially observed widening in Greek spreads only reflected an increase in perceived, but not actual credit risk? Under this alternate premise, Figure 9 suggests that markets would have effectively driven Greece into default with the initial increase in yields triggering an adverse feedback loop between rising yields and deteriorating macroeconomic fundamentals.

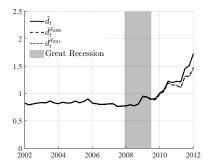
To lend further support to the Greek crisis' self-reinforcing narrative, one would ideally be able to pinpoint 'the match that lit the fuse'. In this context, since investors rely heavily on external credit risk assessments in the proposed theory, the observation that all three major rating agencies downgraded Greek debt in December of 2009 right when spreads had returned to zero for the first time since the outbreak of the 2008 Financial Crisis serves as a natural candidate for said match. In this spirit, one may then be tempted to interpret Figure 9 as the counterfactual evolution of the Greek state in absence of said downgrades. As will become clear in the third and final counterfactual, such an interpretation would be misguided, namely because the corresponding counterfactual Greek state would have been extremely fragile.

In the third and final counterfactual, which is depicted in Figure 10, I examine how Greek yields would have evolved if perceived credit risk had been given by $\check{\lambda}_t^{c_{\bar{t}}} = 0$ for all $t < \bar{t} + 1$ (until the end of year \bar{t}) while allowing the rating agency to issue model-implied credit risk assessments thereafter. The main takeaway from Figure 10 is that even if credit risk assessments had remained at zero beyond the 2008 Financial Crisis and the Great Recession, only minor fears and/or a small increase in the risk free rate would have been sufficient to trigger a divergence in yields.

In summary, even if the self-reinforcing narrative of the Greek crisis is accepted, the inference that the observed default was caused by international investors or by the rating agencies is largely misguided. In particular, this is because self-reinforcing crises can only occur once a sovereign has maneuvered into financially fragile territories of its fundamental state space. For example, even if perceived Greek credit risk had remained low and counterfactual Greek yields had continued to mirror the risk free rate until late 2009, such as due to an earlier inception of OMT, the corresponding counterfactual Greek state would have been so fragile that an eventual credit event would have been inevitable almost surely. However, to the extent that additional time could have be leveraged to consolidate Greek finances, the presented framework does support Mario Draghi's proposition that "breaking expectations" via outright purchases can constitute an effective measure to address a looming sovereign debt crisis.

Figure 10. Counterfactual C: Cumulative effects of counterfactual credit ratings





Notes: Figure 10 depicts the counterfactual evolution of the Greek state assuming two counterfactual credit ratings scenarios. In the first scenario, perceived credit risk is assumed to remain at zero until the end of 2009 while allowing model-implied risk to be priced in thereafter. Importantly, notice that yields continue to mirror the risk free rate throughout all of 2010 during which they are assessed fairly. This confirms the principal insight from Figure 9, namely that more favorable credit risk assessments in 2009 would have decreased actual credit risk thereafter. However, even under these very favorable conditions, the counterfactual Greek state would have been so fragile that the increase in the risk free rate in early 2011 would have been sufficient to induce an explosion in yields. The second scenario is equivalent to the previous counterfactual except that perceived credit risk remains at zero until the end of 2011, in which case actual contemporaneous credit risk would have been negligible at the beginning of 2012.

5 The present Greek state

Since the credit event in 2012, Greece has been heavily reliant on the favorable terms provided to it by the Greek Loan Facility, the EFSF, the ESM, and the IMF. In concrete terms, the fraction of officially sourced Greek debt has consistently exceeded 80% since 2012, whereas it had stood at 16% and 26% in 2010 and 2011 respectively. In anticipation of a future return to market-based

finance, investors might naturally wonder whether the country's current state would be financially sustainable under a counterfactual return to private financing today.

To assess the current Greek state, first note that the current face value of Greek debt — roughly 175% of GDP in Q3 of 2022 — is higher than it was in early 2012. However, due to an increased average maturity — from 6.3 years in 2011 to 17.2 years in 2023 — the present value of said debt is lower. Indeed, given the long average maturity, the present value is highly sensitive to the choice of discount rate (see Figure 11). For example, applying a discount rate of 3% (current German 15Y) yields a present value of 105% of GDP. To the extent that such a discount rate may be perceived as inappropriate, I perform two robustness checks by parameterizing the function f from equation (5) with three separate values of \hat{d}_{2023}^i , each recovered using a different discount rate $\delta_i \in \{0.01, 0.03, 0.05\}$ and an average maturity of 17.2 years.

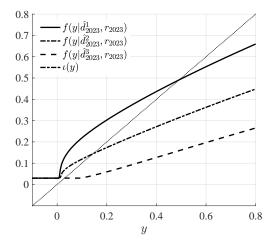


Figure 11. Assessing the current state $(\hat{d}_{2023}, r_{2023})$

Notes: Figure 11 parameterizes the transition function f from equation (5) with current macroeconomic fundamentals $\hat{d}_{2023} \in \{1.47, 1.05, 0.75\}, r_{2023} = 0.03$ and $\hat{\theta}$.

The main insight from Figure 11 is that the recent rise in risk free rates has lead to a bifurcation, whereby the 'good' low-interest-rate steady state was lost for the two more conservative discount rates, i.e. $\delta_i \in \{0.01, 0.03\}$. Thus, so long as markets are using the German 15Y to calculate the present value of Greek debt ($\delta_i = 0.03$), the market dynamics associated with a predominately private financing scheme would lead to a steady state of roughly 15%. In turn, under the even more conservative, too conservative perhaps, scenario ($\delta_i = 0.01$), the corresponding steady state would be close to 50%. In either case, based on the presented analysis, the Greek state of sovereign

finance remains highly fragile such that a quick return to a predominantly private financing scheme is hardly advisable.

6 Conclusion

The presented analysis suggests that an earlier inception of OMT in late 2009 may well have sufficed to materially and positively alter the observed Greek path depicted in Figure 1. Through the lens of the proposed theory, this is because the initial rise in late 2009 set in motion a self-reinforcing yield spiral that OMT plausibly could have prevented. However, even in the counterfactual event that credit risk had remained low, the corresponding counterfactual Greek state's lacking resilience to exogenous perturbations would almost surely have led to another credit event shortly thereafter unless counterfactual policy had simultaneously strengthened Greek macroeconomic fundamentals. This result reflects the conventional wisdom that unconventional monetary policy can complement, but not substitute for sound fiscal policy. Nevertheless, to the extent that additional time could have been leveraged for purposes of fiscal consolidation, the self-reinforcing narrative of the Greek crisis supports former ECB-Chair Draghi's proposition that 'breaking expectations', such as through outright purchases, can serve as an effective measure to combat a looming sovereign debt crisis.

In terms of the status quo, I find that the current Greek state is financially unsustainable in the sense that a quick return to a predominantly private financing scheme would likely lead to a gradual rise of Greek yields to levels in excess of ten percent.

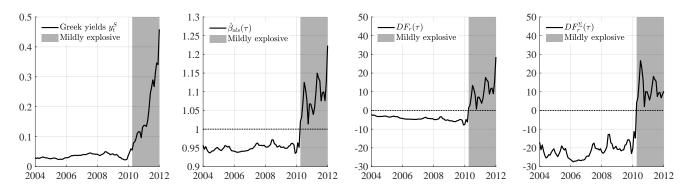
Mildly explosive behavior

To identify "mildly explosive" behavior, Phillips and Yu (2011) propose a set of right-sided unit root tests under the null that a time series is generated by a Gaussian AR(1), a linear model.³³ Specifically, taking the first τ elements of a series with a total number of observations T, Phillips and Yu (2011) construct two sequences of estimators defined as,

$$DF_r(\tau) \equiv \tau [\hat{\beta}_{ols}(\tau) - 1], \qquad DF_r^{\Sigma}(\tau) \equiv \hat{\Sigma}^{\frac{1}{2}}(\tau) [\hat{\beta}_{ols}(\tau) - 1]$$

where $\hat{\beta}_{ols}(\tau)$ is the ordinary-least-squares slope estimate of the AR(1) and $\hat{\Sigma}(\tau) \equiv \sum_{i=1}^{\tau} (y_{i-1}^S - y_{i-1}^S)$ $\overline{y^S})^2/\hat{\sigma}^2$. Here, it is exploited that under $H_0: \beta=1$, both estimators weakly converge to a well defined distribution as $\tau \to \infty$ (Phillips, 1987). Proceeding as such, as illustrated in Figure 12, both estimators reject $H_0: \beta=1$ against $H_1: \beta>1$ at a confidence level of 5% starting in March 2010.

Figure 12. Right-sided unit-root test (see Philips and Yu, 2011)



Notes: Figure 12 depicts the evolution of $\hat{\beta}_{ols}(\tau)$, $DF_r(\tau)$ and $DF_r^{\Sigma}(\tau)$) as τ more observation are included in the regression. As can be seen, the proposed procedure rejects $H_0: \beta = 1$ against $H_1: \beta > 1$ starting in March of 2010. This assessment is unsurprising insofar as the identified, explosive behavior is clearly visible in subfigure a).

Rejecting $H_0: \beta = 1$ against $H_1: \beta > 1$ need not imply that $H_1: \beta > 1$ is particularly sensible hypothesis. Specifically, recall that we have assumed that our process is AR(1). Examining the original yield series, it is evident that if the data is indeed generated by an AR(1), it is not credible that β exceeded unity over the entire observed interval.³⁴ Accordingly, the only remaining rationale that can save our *linear premise* is that the observed kink in Greek yields was in fact caused by a perturbation of the parameter β , or a structural break.

³³Specifically, we have $y_{t+1}^S = \mu_y + \beta y_t^S + \varepsilon_{t+1}^y$ with $\varepsilon_{t+1}^y \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_y^2)$.

³⁴An analysis of the model-implied distribution of required shocks — virtually all negative between 2004 and 2010 soundly rejects the zero-mean assumption in a local sense (as evidenced by $\hat{\beta}_{ols} < 1$ for that time period).

B Uncovering the evolution of \check{y}_t

Figure 13 explores the origins of the the rapid, explosive rise in the asymptotic fixed point $\mathcal{P}(\tilde{d}_t, r_t)$ between 2008 and 2012 as shown in Figure 8. For this, I parameterize the difference equation (5) with the observed, fourth-quarter macroeconomic fundamentals — external financing need and risk free interest rate — from 2008, 2009, 2010, and 2011 respectively.

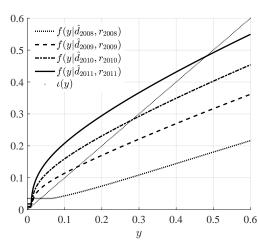


Figure 13. Origins of $\mathcal{P}(\tilde{d}_t, r_t)$

Notes: Figure 13 illuminates the origins of \check{y}_t by plotting the evolution of f against the identity function ι . The four intersections represent the four fixed points towards which the parameterized system would have converged if Greek debt levels and the risk free interest rate had remained at their respective contemporaneous (fourth quarter) levels.

The main takeaway from Figure 13 is then that the deterioration in macroeconomic fundamentals following the 2008 Financial Crisis caused a continued upward tilt in f, which in turn gave rise to ever higher, or 'worse', asymptotic fixed points.

C Strategic default

The payoff table of the default game is motivated as follows. First, the government strictly prefers to remain in power, whereas the political base strictly prefers no default. Conditional on remaining in power, the government prefers to default if and only if $e > \underline{e}$. Conditional on not being reelected, the government would prefer no default such that the incoming delegation inherits the present fiscal imbalance, thus making the current administration look more favorably ex post. Conditional on default, the political base prefers to punish the present government by not reelecting

it. Finally, conditional on no default, the political base prefers to reelect the government.³⁵

The following table depicts the situation, in which we have $e > \underline{e}$ such that the government prefers to default conditional on being reelected.

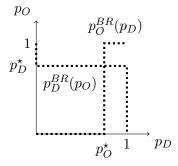
Policy maker

$$(b_3,g_4)$$
 (b_2,g_3) Not reelected Reelect

Default Not Default

Clearly, since $b_i > b_j$, $g_i > g_j$ for each pair (i, j) with i < j, the above game does not feature a Nash equilibrium in pure strategies. Instead, letting $p_D \equiv \Pr(\sigma^G = \text{default})$ and $p_O = \Pr(\sigma^B = \text{not reelect})$, unique equilibrium is given by the tuple of mixed-strategies depicted in Figure 14.

Figure 14. Equilibrium $(p_D^{\star}, p_O^{\star})$ in the political game



D A particle filter

I estimate the parameter vector θ by approximating the Bayesian posterior distribution $\Pr(\theta|Y^T)$,

$$\theta_{\text{MAP}} \equiv \operatorname{argmax}_{\theta \in \Theta} \ \Pr(\theta | Y^T)$$

$$\theta_{\text{BPM}} \equiv \int_{\Theta} \theta \Pr(\theta | Y^T) d\theta$$

where $\Pr(\theta|Y^T)$ α $\mathcal{L}(Y^T|\theta)$ $\Pr(\theta)$, $\mathcal{L}(Y^T|\theta)$ is the true likelihood of the data Y^T , and $\Pr(\theta)$ is the joint prior. The approximate sample analogues are then given by $\Pr(\theta|Y^T)$ and $\hat{\mathcal{L}}(Y^T|\theta)$.

To construct the likelihood $\hat{\mathcal{L}}$, I use equation (1') to forward propagate a swarm of old state particle $(y_{it-1}^S, r_{it}, \tilde{d}_{it})$ with random draws of the shock ε_t^{ξ} and measurement errors η_t^d . As discussed

³⁵The simultaneous nature of the game is motivated by the idea that both elections and orderly defaulting require some preparation.

in the main text, we have,

$$X_t = F(X_{t-1}, \varepsilon_t | \theta), \quad Y_t = X_t + \eta_t$$

where $X_t = (y_{t-1}^S, r_t, d_t)$, $\varepsilon_t = (\varepsilon_t^{\xi}, \varepsilon_t^r, \varepsilon_t^d)$, $Y_t = (\hat{y}_{t-1}^S, \hat{r}_t, \hat{d}_t)$, and $\eta_t = (0, 0, \eta_t^d)$ denote the unobserved state, the observables, a vector of fundamental shocks, and a vector of measurement errors. Since the first two elements of η_t are zero, I explicitly assume that the risk free interest rate and Greek yields are observed without measurement error.

In contrast to the classical particle filter advertised by Fernández-Villaverde and Rubio-Ramírez (2007), in which particles are generated by simulating shocks and weighting occurs via the joint density of measurement errors, I instead simulate measurement errors and 'backward engineer' the model-implied shock. Before discussing why this approach is preferred, I first show that it is valid. For this, notice that we can decompose the likelihood as follows,

$$\begin{split} P(Y^T|\theta) &= \Pr(Y_T|Y^{T-1};\theta) \Pr(Y^{T-1}|\theta) \\ &= \prod_{t=1}^T \Pr(Y_t|Y^{t-1}) \\ &= \prod_{t=1}^T \iint \underbrace{\Pr(Y_t|\varepsilon_t, X_0, Y^{t-1};\theta)}_{\text{Weigh particle}} \underbrace{\Pr(\varepsilon_t, X_0|Y^{t-1};\theta)}_{\text{Simulate particle}} d\varepsilon_t dX_0 \\ &= \prod_{t=1}^T \iint \underbrace{\Pr(Y_t|\eta_t, X_0, Y^{t-1};\theta)}_{\text{Weigh particle}} \underbrace{\Pr(\eta_t, X_0|Y^{t-1};\theta)}_{\text{Simulate particle}} d\eta_t dX_0 \end{split}$$

where the last equality illustrates that we can principally simulate our particles by either drawing shocks or measurement errors (or even a mixture of the two) as long as our approach yields a distribution over the observation Y_t that we can evaluate with the remaining densities. This condition is satisfied in the canonical setup, in which all shocks are simulated and all observations allow for measurement error. In my case, however, the only endogenous state is observed without error. To generate the likelihood, I must thus either simulate a shock $\tilde{\varepsilon}_{it}^{\xi}$ and backward engineer a measurement error $\tilde{\eta}_{it}^d$ that rationalizes the new state y_t^S or I can alternatively simulate a measurement error $\tilde{\eta}_{it}^d$ and backward engineer a corresponding shock $\tilde{\varepsilon}_{it}^{\xi}$. The reason why the latter approach is contextually more appropriate is that the model-implied error-to-shock mapping is analytically available, whereas the inverse shock-to-error mapping is not. I thus choose to simulate measure-

ment errors and backward engineer the model-implied shock because this approach allows me to exploit the closed-form nature of (1') without having to resolve for equilibrium for each parameter candidate anew. Concretely, we thus have,

$$\mathcal{L}(Y^{T}|\theta) \equiv \Pr(Y^{T}|\theta)$$

$$\simeq \prod_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} \Pr(Y_{t}|\eta_{it}, x_{i0}, Y^{t-1}; \theta)$$

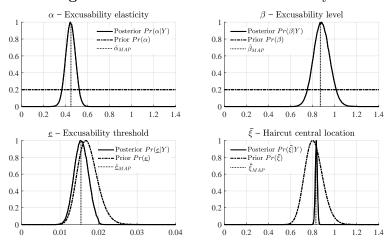
$$\simeq \prod_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} f_{\varepsilon}(\varepsilon_{it}|\eta_{it}, x_{i0}, Y^{t-1}; \theta)$$

$$= \prod_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} f_{\varepsilon\xi}(\hat{\varepsilon}_{it}^{\xi})[1 - \tilde{\pi}_{it}] \equiv \hat{\mathcal{L}}(Y^{T}|\theta)$$

where $\tilde{\varepsilon}_{it}^{\xi}$ and $\tilde{\pi}_{it}$ are the loss-given-default shock weighed by its density $f_{\varepsilon\xi}$ and the probability of default implied by the particle $\tilde{\eta}_{it}^d$. To ensure smoothness of the likelihood over the parameter space, I recycle the same measurement errors for each evaluation of $\hat{\mathcal{L}}$.

To recover $\hat{\theta}_{MAP}$, I choose priors $\Pr(\theta)$ and maximize the resulting approximate posterior distribution $\Pr(\theta|Y^T)$ α $\hat{\mathcal{L}}(Y^T|\theta)\Pr(\theta)$ with the particleswarm routine native to Matlab. I then iteratively sample one million draws from $\Pr(\theta|Y^T)$ using a Metropolis algorithm (initialized at $\hat{\theta}_{MAP}$) and plot the resulting empirical posterior against its priors in Figure 15.

Figure 15. Prior vs. Posterior Probability



Notes: Figure 15 plots the posterior density of each parameter against its chosen prior. For α and β , an uninformative prior is chosen so as to effectively only impose $\alpha, \beta > 0$. For \underline{e} , I choose a lognormal with mode $\frac{1}{60}$, which corresponds — since Greece historically has a tax-to-GDP ratio of one third — to a target of roughly 5% of tax income. In turn, the standard deviation is set to target an effective lower bound of $\frac{1}{100}$, which corresponds to roughly 3% of tax income. Finally, the prior for $\bar{\xi}$ is chosen to target a mode of 0.8 with an effective lower bound of 0.6.

 $^{^{36}}$ Here, the notion of an 'effective lower bound' captures the idea that posterior probability is low by construction

Finally, I calculate $\hat{\beta}_{BPM}$ by taking the sample average along all dimensions of my empirical posterior. The posterior means $\hat{\beta}_{BPM}$ and $\hat{\xi}_{BPM}$ weakly exceed $\hat{\beta}_{MAP}$ and $\hat{\xi}_{MAP}$ which suggests that those distributions are right-skewed. Conversely, $\hat{\alpha}_{MAP}$ weakly exceeds $\hat{\beta}_{BPM}$ such that the posterior of α is left-skewed. All counterfactuals are conducted with both candidate estimates to confirm that the results are indeed invariant to the choice in posterior mode versus posterior mean, which they are.

E Data

The data sources for all series displayed in Figure 1 are shown in Table 2

TABLE 2. PRIMARY DATA

	Series	Source
Fig. 1A	3Y Greek yields 3Y German yields	Monthly, via Bank of Greece Quarterly, via Investing.com
Fig. 1B	Credit Ratings	Aperiodic, via Trading Economics
Other	Weighted av. maturity	Annual, via Greek Public Debt Management Agency

The model is parameterized using quarterly data from 2001:I until December of 2011:IV. Before considering my employed observations — \hat{y}_{t-1}^S , \hat{r}_t , and \hat{d}_t — in Table 3, it is important to note that real-world countries do not periodically refinance their entire stock of debt. This brings about two complications when taking my model to the data.

First, I must select a specific maturity to represent y_t^S . I choose a remaining maturity of three years (3Y) for the primary reason that it allows me to pick up credit risk fears well in advance while simultaneously also reflecting fears at much shorter horizons as evidenced by the fact that 3Y yields continued to rise when default was already imminent. This is because, as accurately anticipated by markets, holders of virtually all horizons were bailed in as part of the 2012 debt restructuring (as opposed to a sequential skipping of payments whenever they come due).³⁷

The second issue that arises from overlapping finance derives from the fact that measured.

unless the likelihood were to be extremely informative with a very high local mass.

³⁷The main point here is that haircuts may be applied to bonds before they mature. Therefore, a bond with a remaining maturity of three years may still very well be subject to default within a quarter. Intuitively, the proposed framework is thus best interpreted as featuring a government which — rather than issuing additional bonds to finance its contemporaneous spending — refinances its entire portfolio with a new three year loan each quarter.

maturity-unadjusted debt-to-GDP values do not coincide with the modeled values such that recovering a real-world equivalent of the 'external financing need' d_t is nontrivial. Specifically, recall that d_t quantifies the burden of all presently outstanding debt by computing the liquidity that would be needed to settle all outstanding claims now and thus with certainty. Therefore, d_t does not represent the canonical present value of the observed Greek debt — which accounts for default — but rather the present value of all outstanding claims in absence of default.³⁸ In effect, I calculate \hat{d}_t by discounting the observed per-GDP face value (FV) using weighted average maturities and the corresponding risk free rate. To mitigate concerns regarding the precision of $\{\hat{d}_t\}$, I allow for measurement error when constructing the particle filter.

TABLE 3. OBSERVABLES USED FOR ESTIMATION

	Variable	In Table 2
$ \begin{array}{c} \hat{y}_t^S \\ \hat{r}_t \\ \hat{d}_t \end{array} $	Secondary market yield Risk free rate External Financing Need	Quarterly, 3Y Greek yield Quarterly, min{3Y German, 3Y Greece} Quarterly, discounted FV of debt/GDP

Notes: External financing needs \hat{d}_t are calculated by discounting the observed per-GDP face value of Greek debt \hat{FV}_t (via Eurostat) with the weighted average maturity of all outstanding debt \hat{m}_t and a correspondingly interpolated German Bund rate $\hat{r}_t^{\hat{m}}$ (e.g. for a remaining maturity of six years, I linearly interpolate between the German 5Y and 7Y): $\hat{d}_t = \hat{FV}_t/(1+\hat{r}_t^{\hat{m}})^{\hat{m}_t}$.

In constructing my empirical counterfactuals, I require additional data on nominal GDP growth and use it to recover the model-implied primary deficit using the identity $x_t = d_t - \frac{V_t}{1+g_t}$ and the smoothed state estimate $(\tilde{d}_t, \tilde{V}_t)$.

TABLE 4. SECONDARY DATA (USED FOR COUNTERFACTUALS)

	Theory	Data equivalent
\hat{g}_t \hat{x}_t	Nominal GDP growth Primary deficit	Quarterly, via Eurostat Quarterly, model-implied

Notes: Nominal GDP growth \hat{g}_t is directly computed from the data while the primary deficit measure \hat{x}_t is constructed using the model-implied identity $\hat{x}_t = \tilde{d}_t - \frac{\tilde{V}_t}{1+\hat{g}_t}$.

³⁸To the debtor, the burden of having to settle all outstanding claims in present value terms is invariant to current yields. Of course, current yields do affect the new bond's face value that is required to refinance.

TABLE 5. CHRONOLOGY OF THE GREEK SOVEREIGN DEBT CRISIS

Month	Event
October 2008	BoG: "The Greek economy exhibits serious structural weaknesses and chronic imbalances that have remained unaddressed for a protracted period."
February 2009	BoG: "Greece must break with [] a model of overconsumption, sizeable imports, and lasting twin deficits and debts."
October 2009	Greek authorities announce that the 2009 budget deficit is more than double its projection while the 2008 deficit is also revised significantly.
December 2009	All major rating agencies — S&P, Fitch, and Moody's — downgrade Greek debt with Fitch being the first to label it as 'non-investment grade'.
April 2010	Greece officially requests financial support from the European Union (EU) and the International Monetary Fund (IMF).
May 2010	EU and IMF announce separate financial support programs totaling \leq 110 billion. The European Central Bank (ECB) continues to accept Greek bonds as collateral despite non-investment grade ratings.
October 2010	In addition to sluggish Greek tax revenues, concerns in Ireland and Portugal cause sovereign spreads to soar across all peripheral states.
June 2011	The Troika concludes that further reforms are inevitable. The country is plagued by public riots and political instability.
July 2011	Eurozone members agree to new "measures designed to alleviate the Greek debt crisis and ensure the financial stability of the euro area as a whole". Greek debt is downgraded to extremely speculative by all major rating agencies.
September 2011	The Troika abruptly leaves Athens after talks with the Greek government are unsuccessful. Media paint scenarios of a Greek default and Eurozone exit.
October 2011	Private investors are expected to agree to a haircut at a "nominal discount" of 50 percent.
November 2011	Although George Papandreou wins a parliamentary confidence vote, the Prime Minister resigns shortly thereafter.
December 2011	The new Greek government releases its budget plan for 2012. While Greece's fiscal state is predicted to improve, the expected recovery is largely due to the anticipated debt restructuring.
March 2012	After resolving technical and legal issues, the Greek government takes advantage of collective action clauses and successfully restructures its debt.

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